Appendix

1 The Pre-PTA Equilibria

The zero profit conditions for the national firms in countries 1, 2, 3 and multinational firms are:

$$\left(\frac{L_1}{(n_1+m)+(n_2+n_3)\tau^{\sigma-1}}+\frac{L_2\tau^{\sigma-1}}{(n_2+m)+(n_1+n_3)\tau^{\sigma-1}}+\frac{L_3\tau^{\sigma-1}}{(n_3+m)+(n_1+n_2)\tau^{\sigma-1}}\right)\frac{\theta}{\sigma}\leq F_1+F_2 \quad \text{(A.1)}$$

$$\left(\frac{L_1\tau^{\sigma-1}}{(n_1+m)+(n_2+n_3)\tau^{\sigma-1}}+\frac{L_2}{(n_2+m)+(n_1+n_3)\tau^{\sigma-1}}+\frac{L_3\tau^{\sigma-1}}{(n_3+m)+(n_1+n_2)\tau^{\sigma-1}}\right)\frac{\theta}{\sigma}\leq F_1+F_2 \quad \text{(A.2)}$$

$$\left(\frac{L_1\tau^{\sigma-1}}{(n_1+m)+(n_2+n_3)\tau^{\sigma-1}}+\frac{L_2\tau^{\sigma-1}}{(n_2+m)+(n_1+n_3)\tau^{\sigma-1}}+\frac{L_3}{(n_3+m)+(n_1+n_2)\tau^{\sigma-1}}\right)\frac{\theta}{\sigma}\leq F_1+F_2 \quad \text{(A.3)}$$

$$\left(\frac{L_1}{(n_1+m)+(n_2+n_3)\tau^{\sigma-1}}+\frac{L_2}{(n_2+m)+(n_1+n_3)\tau^{\sigma-1}}+\frac{L_3}{(n_3+m)+(n_1+n_2)\tau^{\sigma-1}}\right)\frac{\theta}{\sigma}\leq F_1+3F_2 \quad \text{(A.4)}$$

$$n_1, n_2, n_3, m \geq 0$$

A.1 When The Pre-PTA Equilibria Are Mixed

Case I: $n_1 > 0, n_2 > 0, n_3 > 0, m > 0$

In this equilibrium, (A.1)-(A.4) are all equalities. Solving (A.1)-(A.3), we get:

$$\frac{L_1}{(n_1+m)+(n_2+n_3)\tau^{\sigma-1}}=\frac{L_2}{(n_2+m)+(n_1+n_3)\tau^{\sigma-1}}=\frac{L_3}{(n_3+m)+(n_1+n_2)\tau^{\sigma-1}} \quad \text{(A.5)}$$

We can derive the following from (A.5):

$$2\tau^{\sigma-1}<\frac{L_I}{L_3}<\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \quad \text{(A.6)}$$

Solving (A.1)-(A.4), we get:

$$\left(\frac{L_1}{(n_1+m)+(n_2+n_3)\tau^{\sigma-1}}\right)\frac{\theta}{\sigma}=\left(\frac{L_2}{(n_2+m)+(n_1+n_3)\tau^{\sigma-1}}\right)\frac{\theta}{\sigma}=\left(\frac{L_3}{(n_3+m)+(n_1+n_2)\tau^{\sigma-1}}\right)\frac{\theta}{\sigma}$$

$$= \frac{F_2}{1 - \tau^{\sigma-1}} \tag{A.7}$$

Substituting (A.7) in (A.1), we get following condition for this equilibrium:

$$\frac{F_1}{F_2} = \frac{3\tau^{\sigma-1}}{1 - \tau^{\sigma-1}} \tag{A.8}$$

Case II: $n_1 > 0, n_2 > 0, n_3 = 0, m > 0$ In this case, (A.1), (A.2) and (A.4) are equalities whereas (A.3) is an inequality as there are no national firms in country 3. Subtracting (A.2) from (A.1), We get:

$$\frac{L_1}{(n_1 + m) + (n_2)\tau^{\sigma-1}} = \frac{L_2}{(n_2 + m) + (n_1)\tau^{\sigma-1}} \tag{A.9}$$

$$n_1 = \left[\frac{L_1 - L_2\tau^{\sigma-1}}{L_2 - L_1\tau^{\sigma-1}}\right] n_2 + \left[\frac{L_1 - L_2}{L_2 - L_1\tau^{\sigma-1}}\right] m \tag{A.10}$$

Subtracting (A.1) from (A.4), we get:

$$\frac{\theta}{\sigma}\left[\frac{L_2}{(n_1 + m) + (n_2)\tau^{\sigma-1}}\right] + \frac{\theta}{\sigma}\left[\frac{L_3}{(m) + (n_1 + n_2)\tau^{\sigma-1}}\right] = \frac{2F_2}{1 - \tau^{\sigma-1}} \tag{A.11}$$

Substituting (A.11) in (A.1):

$$\frac{\theta}{\sigma}\left[\frac{L_1}{(n_1 + m) + (n_2)\tau^{\sigma-1}}\right] = \frac{F_2(1 - 3\tau^{\sigma-1}) + F_1(1 - \tau^{\sigma-1})}{1 - \tau^{\sigma-1}} \tag{A.12}$$

Substituting for n_1 from (A.10) and solving for n_2:

$$n_2 = \left[\frac{\theta}{\sigma}\left(\frac{L_2 - L_1\tau^{\sigma-1}}{\left[F_2(1 - 3\tau^{\sigma-1}) + F_1(1 - \tau^{\sigma-1})\right](1 + \tau^{\sigma-1})}\right) - \frac{m}{(1 + \tau^{\sigma-1})}\right] \tag{A.13}$$

Subtracting (A.2) from (A.4):

$$\frac{\theta}{\sigma}\left[\frac{L_1}{(n_1 + m) + (n_2)\tau^{\sigma-1}}\right] = \frac{2F_2}{1 - \tau^{\sigma-1}} - \frac{\theta}{\sigma}\left[\frac{L_3}{(m) + (n_1 + n_2)\tau^{\sigma-1}}\right] \tag{A.14}$$

Substituting (A.14) in (A.1):

$$\frac{\theta}{\sigma}\left[\frac{L_3}{(m)+(n_1+n_2)\tau^{\sigma-1}}\right]=\frac{F_2(1+3\tau^{\sigma-1})-F_1(1-\tau^{\sigma-1})}{1-\tau^{\sigma-1}} \tag{A.15}$$

Substituting for n_1 and solving for n_2:

$$n_2=\left[\frac{\theta}{\sigma}\left(\frac{L_3(L_2-L_1\tau^{\sigma-1})}{\left[F_2(1+3\tau^{\sigma-1})-F_1(1-\tau^{\sigma-1})\right](L_1+L_2)\tau^{\sigma-1}}\right)-\frac{L_2}{(L_1+L_2)\tau^{\sigma-1}}m\right] \tag{A.16}$$

Solving for m from (A.13) and (A.16)

$$\left[\frac{1}{(1+\tau^{\sigma-1})}-\frac{L_2}{(L_1+L_2)\tau^{\sigma-1}}\right]m=$$

$$=\frac{\theta}{\sigma}(L_2-L_1\tau^{\sigma-1})\left[A-B\right] \tag{A.17}$$

Where[1]

$$A=\frac{1}{\left[F_2(1-3\tau^{\sigma-1})+F_1(1-\tau^{\sigma-1})\right](1+\tau^{\sigma-1})}$$

$$B=\frac{L_3}{\left[F_2(1+3\tau^{\sigma-1})-F_1(1-\tau^{\sigma-1})\right](L_1+L_2)\tau^{\sigma-1}}$$

$$\left[\frac{1}{(1+\tau^{\sigma-1})}-\frac{L_2}{(L_1+L_2)\tau^{\sigma-1}}\right]=\frac{L_1\tau^{\sigma-1}-L_2}{(1+\tau^{\sigma-1})(L_1+L_2)\tau^{\sigma-1}}<0$$

The Left hand side of (A.17) is always negative implying that the right hand side should be negative too. Setting right hand side equal to zero gives us the following condition for this equilibrium to exist:

$$\frac{L_I\tau^{\sigma-1}(1+3\tau^{\sigma-1})-L_3(1-3\tau^{\sigma-1})(1+\tau^{\sigma-1})}{L_I(1-\tau^{\sigma-1})\tau^{\sigma-1}+L_3(1-\tau^{2(\sigma-1)})}<\frac{F_1}{F_2} \tag{A.18}$$

[1]We can write (A.9) as $n_1(L_1\tau^{\sigma-1}-L_2)+m(L_1-L_2)+n_2(L_1-L_2\tau^{\sigma-1})=0$. If $L_1 \gtrless L_2 \implies L_1\tau^{\sigma-1}-L_2<0$ or $L_2-L_1\tau^{\sigma-1}>0$

Solving (A.13) and (A.16) for m:

$$m = \left[\frac{\theta}{\sigma}\left(\frac{L_2 - L_1\tau^{\sigma-1}}{[F_2(1-3\tau^{\sigma-1}) + F_1(1-\tau^{\sigma-1})]}\right) - (1+\tau^{\sigma-1})n_2\right] \tag{A.19}$$

$$m = \left[\frac{\theta}{\sigma}\left(\frac{L_3(L_2 - L_1\tau^{\sigma-1})}{[F_2(1+3\tau^{\sigma-1}) - F_1(1-\tau^{\sigma-1})]\,L_2}\right) - \frac{(L_1+L_2)\tau^{\sigma-1}n_2}{L_2}\right] \tag{A.20}$$

Equating (A.19) and (A.20) and solving for n_2:

$$n_2 = \frac{\theta}{\sigma}\left[\frac{L_2}{[F_2(1-3\tau^{\sigma-1}) + F_1(1-\tau^{\sigma-1})]} - \frac{L_3}{[F_2(1+3\tau^{\sigma-1}) - F_1(1-\tau^{\sigma-1})]}\right] \tag{A.21}$$

Since the left hand side is positive, we set the right hand side to be positive and get:

$$\frac{(L_2+L_3)(1+3\tau^{\sigma-1}) - 2L_3}{(L_2+L_3)(1-\tau^{\sigma-1})} > \frac{F_1}{F_2} \tag{A.22}$$

We can, similarly, get two equations for n_1 and find the following condition:

$$\frac{(L_1+L_3)(1+3\tau^{\sigma-1}) - 2L_3}{(L_1+L_3)(1-\tau^{\sigma-1})} > \frac{F_1}{F_2} \tag{A.23}$$

Adding (A.22) and (A.23), we get the following condition for this equilibrium:

$$\frac{L_I(1+3\tau^{\sigma-1}) - 2L_3(1-3\tau^{\sigma-1})}{(L_I+2L_3)(1-\tau^{\sigma-1})} > \frac{F_1}{F_2} \tag{A.24}$$

Subtracting (A.3) from (A.4) and substituting (A.9):

$$\frac{\theta}{\sigma}\left[\frac{L_1}{(n_1+m) + (n_2)\tau^{\sigma-1}}\right] > \frac{F_2}{1-\tau^{\sigma-1}} \tag{A.25}$$

Substituting (A.15) and (A.25) in (A.1):

$$\frac{F_1}{F_2} > \frac{3\tau^{\sigma-1}}{1-\tau^{\sigma-1}} \tag{A.26}$$

Case III: $n_1 = 0, n_2 = 0, n_3 > 0, m > 0$

Subtracting (A.3) from (A.4):

$$\frac{\theta}{\sigma}\left[\frac{(L_1+L_2)}{(m)+(n_3)\tau^{\sigma-1}}\right]=\frac{2F_2}{1-\tau^{\sigma-1}} \tag{A.27}$$

Substituting (A.27) in (A.3):

$$\frac{\theta}{\sigma}\left[\frac{L_3}{m+n_3}\right]=\frac{F_2(1-3\tau^{\sigma-1})+F_1(1-\tau^{\sigma-1})}{1-\tau^{\sigma-1}} \tag{A.28}$$

Adding (A.1) and (A.2) and substitute (A.27) and (A.28):

$$\frac{F_1}{F_2}>\frac{3\tau^{\sigma-1}}{1-\tau^{\sigma-1}} \tag{A.29}$$

Rewriting (A.27) and (A.28):

$$m=\frac{\theta}{\sigma}\left[\frac{L_3(1-\tau^{\sigma-1})}{F_2(1-3\tau^{\sigma-1})+F_1(1-\tau^{\sigma-1})}\right]-n_3 \tag{A.30}$$

$$m=\frac{\theta}{\sigma}\left[\frac{(L_1+L_2)(1-\tau^{\sigma-1})}{2F_2}\right]-n_3\tau^{\sigma-1} \tag{A.31}$$

Equating (A.30) and (A.31):

$$n_3=\frac{\theta}{\sigma}\left[\frac{L_3}{F_2(1-3\tau^{\sigma-1})+F_1(1-\tau^{\sigma-1})}-\frac{(L_1+L_2)}{2F_2}\right] \tag{A.32}$$

Since the left hand side is positive, the following condition should be satisfied:

$$\frac{2L_3-L_I(1-3\tau^{\sigma-1})}{L_I(1-\tau^{\sigma-1})}>\frac{F_1}{F_2} \tag{A.33}$$

Similarly, rewriting (A.30) and (A.31) in terms of m and noting that $m>0$, we get:

$$\frac{2L_3\tau^{\sigma-1}-L_I(1-3\tau^{\sigma-1})}{L_I(1-\tau^{\sigma-1})}<\frac{F_1}{F_2} \tag{A.34}$$

A.2 When The Pre-PTA Equilibria Are Pure Multinational

Case I: $n_1 = 0, n_2 = 0, n_3 = 0, m > 0$

Since (A.4) will be an equality in this case, we solve for m:

$$m = \frac{\theta}{\sigma}\left(\frac{L_1 + L_2 + L_3}{F_1 + 3F_2}\right) \tag{A.35}$$

Substituting (A.35) in (A.3):

$$\left(\frac{(L_1 + L_2)\tau^{\sigma-1} + L_3}{L_1 + L_2 + L_3}\right) < \frac{F_1 + F_2}{F_1 + 3F_2} \tag{A.36}$$

$$\frac{2L_3 - L_I(1 - 3\tau^{\sigma-1})}{L_I(1 - \tau^{\sigma-1})} < \frac{F_1}{F_2} \tag{A.37}$$

Similarly, substituting (A.35) in (A.1) and (A.2):

$$\frac{2L_1 - (L_2 + L_3)(1 - 3\tau^{\sigma-1})}{(L_2 + L_3)(1 - \tau^{\sigma-1})} < \frac{F_1}{F_2} \tag{A.38}$$

$$\frac{2L_2 - (L_1 + L_3)(1 - 3\tau^{\sigma-1})}{(L_1 + L_3)(1 - \tau^{\sigma-1})} < \frac{F_1}{F_2} \tag{A.39}$$

Adding (A.38) and (A.39):

$$\frac{L_I(1 + 3\tau^{\sigma-1}) - 2L_3(1 - 3\tau^{\sigma-1})}{(L_I + 2L_3)(1 - \tau^{\sigma-1})} < \frac{F_1}{F_2} \tag{A.40}$$

A.3 When The Pre-PTA Equilibria Are Exporting

Case I: $n_1 > 0, n_2 > 0, n_3 > 0, m = 0$

Subtracting (A.2) from (A.1) and subtracting (A.3) from (A.1):

$$\frac{L_1}{(n_1) + (n_2 + n_3)\tau^{\sigma-1}} = \frac{L_2}{(n_2) + (n_1 + n_3)\tau^{\sigma-1}} = \frac{L_3}{(n_3) + (n_1 + n_2)\tau^{\sigma-1}} \tag{A.41}$$

$$n_1(L_1\tau^{\sigma-1} - L_3) + n_2(L_1\tau^{\sigma-1} - L_3\tau^{\sigma-1}) + n_3(L_1 - L_3\tau^{\sigma-1}) = 0 \tag{A.42}$$

$$n_2(L_2\tau^{\sigma-1} - L_3) + n_1(L_2\tau^{\sigma-1} - L_3\tau^{\sigma-1}) + n_3(L_2 - L_3\tau^{\sigma-1}) = 0 \tag{A.43}$$

Add (C.42) and (C.43):

$$(n_{1+}n_2)\left[(L_1 + L_2)\tau^{\sigma-1} - L_3(1 + \tau^{\sigma-1})\right] = n_3\left[2L_3\tau^{\sigma-1} - (L_1 + L_2)\right] \tag{A.44}$$

Since n_1, n_2 and n_3 are all positive, the other two terms in the expression must be of the same sign.[2]

$$2L_3\tau^{\sigma-1} - (L_1 + L_2) < 0 \Longrightarrow \frac{L_1 + L_2}{L_3} > 2\tau^{\sigma-1}$$

$$(L_1 + L_2)\tau^{\sigma-1} - L_3(1 + \tau^{\sigma-1}) < 0 \Longrightarrow \frac{L_1 + L_2}{L_3} < \frac{1 + \tau^{\sigma-1}}{\tau^{\sigma-1}}$$

$$\Longrightarrow 2\tau^{\sigma-1} < \frac{L_1 + L_2}{L_3} < \frac{1 + \tau^{\sigma-1}}{\tau^{\sigma-1}} \tag{A.45}$$

Subtracting (A.3) from (A.4):

$$\frac{\theta}{\sigma}\left(\frac{L_1(1 - \tau^{\sigma-1})}{(n_1) + (n_2 + n_3)\tau^{\sigma-1}} + \frac{L_2(1 - \tau^{\sigma-1})}{(n_2) + (n_1 + n_3)\tau^{\sigma-1}}\right) < 2F_2 \tag{A.46}$$

Using (A.46):

$$\frac{\theta}{\sigma}\left(\frac{L_1}{(n_1) + (n_2 + n_3)\tau^{\sigma-1}}\right) < \frac{F_2}{(1 - \tau^{\sigma-1})} \tag{A.47}$$

Similarly, we can find:

$$\frac{\theta}{\sigma}\left(\frac{L_2}{(n_2) + (n_1 + n_3)\tau^{\sigma-1}}\right) < \frac{F_2}{(1 - \tau^{\sigma-1})} \tag{A.48}$$

$$\frac{\theta}{\sigma}\left(\frac{L_3}{(n_3) + (n_1 + n_2)\tau^{\sigma-1}}\right) < \frac{F_2}{(1 - \tau^{\sigma-1})} \tag{A.49}$$

[2] We can show that expression in brackets on the right has to be negative. Let us suppose it is positive. $2L_3\tau^{\sigma-1} - (L_1 + L_2) > 0 \Longrightarrow L_3(\tau^{\sigma-1} + \tau^{\sigma-1}) - (L_1 + L_2) > 0 \Longrightarrow L_3(1 + \tau^{\sigma-1}) - (L_1 + L_2) > 0 \Longrightarrow L_3(1 + \tau^{\sigma-1}) - (L_1 + L_2)\tau^{\sigma-1} > 0$. This leads to a contradiction. Futhermore, let us now suppose $2L_3\tau^{\sigma-1} - (L_1 + L_2) = 0 \Longrightarrow L_3(1 + \tau^{\sigma-1}) - (L_1 + L_2)\tau^{\sigma-1}$ which is again a contradiction implying $2L_3\tau^{\sigma-1} - (L_1 + L_2) < 0$.

Substituting (A.47),(A.48) and (A.49) in (A.1):

$$\frac{F_1}{F_2} < \frac{3\tau^{\sigma-1}}{1-\tau^{\sigma-1}} \tag{A.50}$$

Case II: $n_1 > 0, n_2 > 0, n_3 = 0, m = 0$

Subtracting (A.2) from (A.1):

$$\frac{L_1}{(n_1) + (n_2)\tau^{\sigma-1}} = \frac{L_2}{(n_2) + (n_1)\tau^{\sigma-1}} \tag{A.51}$$

Rearranging terms:

$$n_1 = \left[\frac{L_1 - L_2\tau^{\sigma-1}}{L_2 - L_1\tau^{\sigma-1}}\right] n_2 \tag{A.52}$$

Subtracting (A.3) and (A.4) from (A.2) and using (A.51):

$$\frac{\theta}{\sigma}\left(\frac{L_3(1-\tau^{\sigma-1})}{(n_1+n_2)\tau^{\sigma-1}}\right) < F_2 \tag{A.53}$$

Using (A.52) and (A.53) to solve for n_1 and n_2:

$$n_2 > \frac{\theta}{\sigma}\left[\frac{L_3}{F_2} \times \frac{(L_2 - L_1\tau^{\sigma-1})}{(L_1+L_2)\tau^{\sigma-1}}\right] \tag{A.54}$$

$$n_1 > \frac{\theta}{\sigma}\left[\frac{L_3}{F_2} \times \frac{(L_1 - L_2\tau^{\sigma-1})}{(L_1+L_2)\tau^{\sigma-1}}\right] \tag{A.55}$$

Substituting n_1 and n_2 and (A.51) in (A.2), we get the following condition for this equilibrium:

$$\frac{L_I\tau^{\sigma-1} - L_3(1-2\tau^{\sigma-1})}{L_3(1-\tau^{\sigma-1})} > \frac{F_1}{F_2} \tag{A.56}$$

Subtracting (A.4) from (A.1):

$$\frac{\theta}{\sigma}\left(\frac{L_2}{(n_2)+(n_1)\tau^{\sigma-1}} + \frac{L_3}{(n_2+n_1)\tau^{\sigma-1}}\right) < \frac{2F_2}{(1-\tau^{\sigma-1})} \tag{A.57}$$

Substituting this back in (A.1):

$$\frac{\theta}{\sigma}\left(\frac{L_1}{(n_1)+(n_2)\tau^{\sigma-1}}\right) > \frac{(1-\tau^{\sigma-1})F_1+(1-3\tau^{\sigma-1})F_2}{(1-\tau^{\sigma-1})} \tag{A.58}$$

Solving for n_1 and n_2 and using (A.52)

$$n_2 < \frac{\theta}{\sigma}\left(\frac{(L_2-L_1\tau^{\sigma-1})}{(1+\tau^{\sigma-1})\left[(1-\tau^{\sigma-1})F_1+(1-3\tau^{\sigma-1})F_2\right]}\right) \tag{A.59}$$

$$n_1 < \frac{\theta}{\sigma}\left(\frac{(L_1-L_2\tau^{\sigma-1})}{(1+\tau^{\sigma-1})\left[(1-\tau^{\sigma-1})F_1+(1-3\tau^{\sigma-1})F_2\right]}\right) \tag{A.60}$$

Putting these back in (A.1), we get another condition for this equilibrium:

$$\frac{L_I\tau^{\sigma-1}(1+3\tau^{\sigma-1})-L_3(1-3\tau^{\sigma-1})(1+\tau^{\sigma-1})}{L_I(1-\tau^{\sigma-1})\tau^{\sigma-1}+L_3(1-\tau^{2(\sigma-1)})} < \frac{F_1}{F_2} \tag{A.61}$$

Substituting for n_1 in (A.1) by using (A.52) and solving for n_2:

$$n_2 = \frac{\theta}{\sigma}\left(\frac{(L_2-L_1\tau^{\sigma-1})}{(1-\tau^{\sigma-1})\tau^{\sigma-1}(F_1+F_2)} \times \frac{(1-\tau^{\sigma-1})L_3+\tau^{\sigma-1}(L_1+L_2)}{(L_1+L_2)}\right) \tag{A.62}$$

$$n_1 = \frac{\theta}{\sigma}\left(\frac{(L_1-L_2\tau^{\sigma-1})}{(1-\tau^{\sigma-1})\tau^{\sigma-1}(F_1+F_2)} \times \frac{(1-\tau^{\sigma-1})L_3+\tau^{\sigma-1}(L_1+L_2)}{(L_1+L_2)}\right) \tag{A.63}$$

Substituting these values in (A.3) and simplifying, we get:

$$\frac{L_I}{L_3} > \frac{(1+\tau^{\sigma-1})}{(1-\tau^{\sigma-1})} \tag{A.64}$$

Case III: $n_1 = 0, n_2 = 0, n_3 > 0, m = 0$

From (A.3), we get:

$$n_3 = \frac{\theta}{\sigma}\left(\frac{(L_1+L_2+L_3)}{(F_1+F_2)}\right) \tag{A.65}$$

Substituting this in (A.1), (A.2) and (A.4):

$$\frac{2L_3\tau^{\sigma-1}-L_I(1-3\tau^{\sigma-1})}{L_I(1-\tau^{\sigma-1})} > \frac{F_1}{F_2} \tag{A.66}$$

$$\frac{L_I}{L_3} < 2\tau^{\sigma-1} \tag{A.67}$$

B The Post-PTA Equilibria

The zero profit conditions for the national firms based in the integrated region,the national firms in country 3 and the multinational firms are:

$$\left(\frac{L_I}{(n_I+m)+n_3'\tau^{\sigma-1}} + \frac{L_3\tau^{\sigma-1}}{(n_3'+m)+n_I\tau^{\sigma-1}}\right)\frac{\theta}{\sigma} \leq F_1 + F_2 \tag{B.1}$$

$$\left(\frac{L_I\tau^{\sigma-1}}{(n_I+m)+n_3'\tau^{\sigma-1}} + \frac{L_3}{(n_3'+m)+n_I\tau^{\sigma-1}}\right)\frac{\theta}{\sigma} \leq F_1 + F_2 \tag{B.2}$$

$$\left(\frac{L_I}{(n_I+m)+n_3'\tau^{\sigma-1}} + \frac{L_3}{(n_3'+m)+n_I\tau^{\sigma-1}}\right)\frac{\theta}{\sigma} \leq F_1 + 2F_2 \tag{B.3}$$

$$n_I, n_3', m \geq 0$$

B.1 When The Post-PTA Equilibria Are Mixed

Case I: $n_I > 0, n_3' > 0, m > 0$

Subtracting (B.1) from (B.3) and (B.2) from (B.3)

$$\left(\frac{L_3}{(n_3'+m)+n_I\tau^{\sigma-1}}\right)\frac{\theta}{\sigma} = \frac{F_2}{1-\tau^{\sigma-1}} \tag{B.4}$$

$$\left(\frac{L_I}{(n_I+m)+n_3'\tau^{\sigma-1}}\right)\frac{\theta}{\sigma} = \frac{F_2}{1-\tau^{\sigma-1}} \tag{B.5}$$

Substitute (B4) and (B.5) in (B.1):

$$\frac{F_1}{F_2} = \frac{2\tau^{\sigma-1}}{1-\tau^{\sigma-1}} \tag{B.6}$$

From (B.4) and (B.5), we get:

$$\frac{L_3}{(n'_3 + m) + n_I\tau^{\sigma-1}} = \frac{L_I}{(n_I + m) + n'_3\tau^{\sigma-1}} \tag{B.7}$$

$$\Longrightarrow \tau^{\sigma-1} < \frac{L_I}{L_3} < \frac{1}{\tau^{\sigma-1}} \tag{B.8}$$

Case II: $n_I > 0, n'_3 = 0, m > 0$

Subtracting (B.1) from (B.3):

$$\left(\frac{L_3}{(m) + n_I\tau^{\sigma-1}}\right)\frac{\theta}{\sigma} = \frac{F_2}{1-\tau^{\sigma-1}} \tag{B.9}$$

Substituting (B.9) in (B.1):

$$\left(\frac{L_I}{n_I + m}\right)\frac{\theta}{\sigma} - \frac{F_1(1-\tau^{\sigma-1}) + F_2(1-2\tau^{\sigma-1})}{(1-\tau^{\sigma-1})} \tag{B.10}$$

Rewriting (B.10) as follows:

$$m = \left(\frac{L_I(1-\tau^{\sigma-1})}{F_1(1-\tau^{\sigma-1}) + F_2(1-2\tau^{\sigma-1})}\right)\frac{\theta}{\sigma} - n_I \tag{B.11}$$

From (B.9), we get:

$$m = \left(\frac{L_3(1-\tau^{\sigma-1})}{F_2}\right)\frac{\theta}{\sigma} - n_I\tau^{\sigma-1} \tag{B.12}$$

Solving for n_I and m from (B.11) and (B.12):

$$n_I = \left(\frac{L_I}{F_1(1-\tau^{\sigma-1}) + F_2(1-2\tau^{\sigma-1})} - \frac{L_3}{F_2}\right)\frac{\theta}{\sigma} \tag{B.13}$$

$$m = \left(\frac{L_3}{F_2} - \frac{L_I\tau^{\sigma-1}}{F_1(1-\tau^{\sigma-1}) + F_2(1-2\tau^{\sigma-1})}\right)\frac{\theta}{\sigma} \tag{B.14}$$

For n_I and m to be greater than zero, the following conditions must be met:

$$\frac{F_1}{F_2} < \frac{L_I - (1-2\tau^{\sigma-1})L_3}{L_3(1-\tau^{\sigma-1})} \tag{B.15}$$

$$\frac{F_1}{F_2} > \frac{L_I \tau^{\sigma-1} - (1 - 2\tau^{\sigma-1})L_3}{L_3(1 - \tau^{\sigma-1})} \tag{B.16}$$

Substituting (B.9) and (B.10) in (B.2), we get another condition for this equilibrium

$$\frac{F_1}{F_2} > \frac{2\tau^{\sigma-1}}{1 - \tau^{\sigma-1}} \tag{B.17}$$

Case III: $n_I = 0, n_3' > 0, m > 0$

Subtracting (B.2) from (B.3):

$$\left(\frac{L_I}{(m) + n_3' \tau^{\sigma-1}}\right) \frac{\theta}{\sigma} = \frac{F_2}{1 - \tau^{\sigma-1}} \tag{B.18}$$

Substituting this back in to (B.2):

$$\left(\frac{L_3}{n_3' + m}\right) \frac{\theta}{\sigma} - \frac{F_1(1 - \tau^{\sigma-1}) + F_2(1 - 2\tau^{\sigma-1})}{(1 - \tau^{\sigma-1})} \tag{B.19}$$

Rewriting (B.18) and (B.19) as follows

$$m = \left(\frac{L_I(1 - \tau^{\sigma-1})}{F_2}\right) \frac{\theta}{\sigma} - n_3' \tau^{\sigma-1} \tag{B.20}$$

$$m = \left(\frac{L_3(1 - \tau^{\sigma-1})}{F_1(1 - \tau^{\sigma-1}) + F_2(1 - 2\tau^{\sigma-1})}\right) \frac{\theta}{\sigma} - n_3' \tag{B.21}$$

Solving for n_3' and m

$$n_3' = \left(\frac{L_3}{F_1(1 - \tau^{\sigma-1}) + F_2(1 - 2\tau^{\sigma-1})} - \frac{L_I}{F_2}\right) \frac{\theta}{\sigma} \tag{B.22}$$

$$m = \left(\frac{L_I}{F_2} - \frac{L_3 \tau^{\sigma-1}}{F_1(1 - \tau^{\sigma-1}) + F_2(1 - 2\tau^{\sigma-1})}\right) \frac{\theta}{\sigma} \tag{B.23}$$

For n_3' and m to be greater than zero, the following conditions must be met:

$$\frac{F_1}{F_2} < \frac{L_3 - (1 - 2\tau^{\sigma-1})L_I}{L_I(1 - \tau^{\sigma-1})} \tag{B.24}$$

$$\frac{F_1}{F_2} > \frac{L_3\tau^{\sigma-1} - (1 - 2\tau^{\sigma-1})L_I}{L_I(1 - \tau^{\sigma-1})} \tag{B.25}$$

Substituting (B.18) and (B.19) in (B.1), we get another condition for this equilibrium:

$$\frac{F_1}{F_2} > \frac{2\tau^{\sigma-1}}{1 - \tau^{\sigma-1}} \tag{B.26}$$

B.2 When The Post-PTA Equilibria Are Multinational

Case I: $n_I = 0, n_3' = 0, m > 0$

Solving for m from (B.3):

$$m = \left(\frac{L_I + L_3}{F_1 + 2F_2}\right)\frac{\theta}{\sigma} \tag{B.27}$$

Substituting (B.27) in (B.1) and (B.2):

$$\frac{F_1}{F_2} < \frac{L_I - (1 - 2\tau^{\sigma-1})L_3}{L_3(1 - \tau^{\sigma-1})} \tag{B.28}$$

$$\frac{F_1}{F_2} < \frac{L_3 - (1 - 2\tau^{\sigma-1})L_I}{L_I(1 - \tau^{\sigma-1})} \tag{B.29}$$

B.3 When The Post-PTA Equilibria Are Exporting

Case I: $n_I > 0, n_3 = 0, m = 0$

Solving for n_I directly from (B.1):

$$n_I = \left(\frac{L_I + L_3}{F_1 + F_2}\right)\frac{\theta}{\sigma} \tag{B.30}$$

Substituting (B.30) in (B.2) and (B.3), we get the following two conditions for this equilibrium:

$$\frac{F_1}{F_2} < \frac{L_I\tau^{\sigma-1} - (1 - 2\tau^{\sigma-1})L_3}{L_3(1 - \tau^{\sigma-1})} \tag{B.31}$$

$$\frac{L_I}{L_3} < \frac{1}{\tau^{\sigma-1}} \tag{B.32}$$

Case II: $n_I = 0, n_3 > 0, m = 0$

Solving for n_3 from (B.2):

$$n_3 = \left(\frac{L_I + L_3}{F_1 + F_2}\right)\frac{\theta}{\sigma} \tag{B.33}$$

Substituting (B.33) in (B.1) and (B.3), we get the following two conditions for this equilibrium:

$$\frac{F_1}{F_2} < \frac{L_3\tau^{\sigma-1} - (1 - 2\tau^{\sigma-1})L_I}{L_I(1 - \tau^{\sigma-1})} \tag{B.34}$$

$$\frac{L_I}{L_3} < \tau^{\sigma-1} \tag{B.35}$$

Case III: $n_I > 0, n_3' > 0, m = 0$

Subtracting (B.1) and (B.2) from (B.3):

$$\left(\frac{L_I}{n_I + n_3'\tau^{\sigma-1}}\right)\frac{\theta}{\sigma} < \frac{F_2}{1 - \tau^{\sigma-1}} \tag{B.36}$$

$$\left(\frac{L_3}{n_I\tau^{\sigma-1} + n_3'}\right)\frac{\theta}{\sigma} < \frac{F_2}{1 - \tau^{\sigma-1}} \tag{B.37}$$

Substituting (B.36) and (B.37) in (B.1):

$$\frac{F_1}{F_2} < \frac{2\tau^{\sigma-1}}{1 - \tau^{\sigma-1}} \tag{B.38}$$

Subtracting (B.2) from (B.1):

$$\frac{L_I}{n_I + n_3'\tau^{\sigma-1}} = \frac{L_3}{n_I\tau^{\sigma-1} + n_3'} \tag{B.39}$$

Rewriting (B.39) as follows:

$$n_3' = \left(\frac{L_3 - L_I\tau^{\sigma-1}}{L_I - L_3\tau^{\sigma-1}}\right)n_I \tag{B.40}$$

For n_I and n_3' to be greater than zero, the following condition must be met:

$$\tau^{\sigma-1} < \frac{L_I}{L_3} < \frac{1}{\tau^{\sigma-1}} \tag{B.41}$$

C The Derivation of the Utility Functions

C.1 The Pre-PTA Utility Function

We derive the utility function in the case when $n_1, n_2, n_3, m > 0$. Other cases can be derived from this general case by making appropriate substitutions. The utility function of a country i in the pre-PTA regime can be given as follows:

$$U_i = Q_i^{\theta} Y_i^{1-\theta} \tag{C.1}$$

$$i, j, k = 1, 2, 3 \quad i \neq j \neq k$$

Where $Q_i = \left(\sum q_i^{\pi}\right)^{\frac{1}{\pi}}$ and $\pi = \frac{\sigma-1}{\sigma}$ and Q_i can be written as

$$Q_i = \left((n_i + m)q_{ii}^{\pi} + n_j q_{ji}^{\pi} + n_k q_{ki}^{\pi}\right)^{\frac{1}{\pi}} \tag{C.2}$$

The quantity of a variety produced in country j and sold in country i and the quantity of a variety produced in country i and sold in country i are:

$$q_{ji} = \frac{p_{ji}^{-\sigma}}{(n_i + m)p_{ii}^{1-\sigma} + (n_j)p_{ji}^{1-\sigma} + n_k p_{ki}^{1-\sigma}} \theta L_i \tag{C.3}$$

$$q_{ii} = \frac{p_{ii}^{-\sigma}}{(n_i + m)p_{ii}^{1-\sigma} + n_j p_{ji}^{1-\sigma} + n_k p_{ki}^{1-\sigma}} \theta L_i \tag{C.4}$$

$$i \neq j \neq k$$

Using $p_{ij} = p_{ik} = p_{ii}/\tau = \frac{\sigma}{\sigma-1}\frac{c}{\tau}$ to substitute for the prices above, we can rewrite Q_i:

$$Q_i = \left(\frac{(n_i + m)\left[\left(\frac{\sigma}{\sigma-1}c\right)^{-\sigma}\right]^{\pi} + n_j \left[\left(\frac{\sigma}{\sigma-1}\frac{c}{\tau}\right)^{-\sigma}\right]^{\pi} + n_k \left[\left(\frac{\sigma}{\sigma-1}\frac{c}{\tau}\right)^{-\sigma}\right]^{\pi}}{\left[(n_i + m)\left(\frac{\sigma}{\sigma-1}c\right)^{1-\sigma} + n_j \left(\frac{\sigma}{\sigma-1}\frac{c}{\tau}\right)^{1-\sigma} + n_k \left(\frac{\sigma}{\sigma-1}\frac{c}{\tau}\right)^{1-\sigma}\right]^{\pi}} (\theta L_i)^{\pi} \right)^{\frac{1}{\pi}} \tag{C.5}$$

Simplifying (C.5):

$$Q_i = \left((n_i + m) + n_j\tau^{\sigma-1} + n_k\tau^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \times \theta L_i \times \frac{\sigma - 1}{\sigma c} \tag{C.6}$$

Since, the utility function is of Cobb-Douglas type, we can write the demand for the numeraire good $Y = (1-\theta)L_i$. Substituting for Y and Q_i in the utility function, we can write the the utility of a representative individual in country i in the pre-PTA case as follows:

$$U_i^{\text{Pre}} = C \times \left(n_i + m + n_j\tau^{\sigma-1} + n_k\tau^{\sigma-1}\right)^{\frac{\theta}{\sigma-1}} \tag{C.7}$$

$$n_i, m, n_j, n_k \geq 0$$

Where $C = \theta^\theta(1-\theta)^{1-\theta}\left(\frac{\sigma-1}{\sigma c}\right)^\theta$; $i, j, k = 1, 2, 3$ and $\quad i \neq j \neq k$.

C.2 The Post-PTA Utility Function

We derive the utility function in the case when $n_I, n_3, m > 0$. The utility function of a country i in the post-PTA regime is:

$$U_i = Q_i^\theta Y_i^{1-\theta} \tag{C.8}$$

$$i = I, 3 \; i \neq j$$

Where Q_i=$\left(\sum q_i^\pi\right)^{\frac{1}{\pi}}$ and $\pi = \frac{\sigma-1}{\sigma}$ and Q_i can be written as

$$Q_i = \left((n_i + m)q_{ii}^\pi + n_k q_{ki}^\pi\right)^{\frac{1}{\pi}} \tag{C.9}$$

The quantity of a variety produced in country j and sold in country i and the quantity of a variety produced in country i and sold in country i are:

$$q_{ji} = \frac{p_{ji}^{-\sigma}}{(n_i + m)p_{ii}^{1-\sigma} + n_k p_{ki}^{1-\sigma}}\theta L_i \tag{C.10}$$

$$q_{ii} = \frac{p_{ii}^{-\sigma}}{(n_i + m)p_{ii}^{1-\sigma} + n_k p_{ki}^{1-\sigma}} \theta L_i \tag{C.11}$$

Using $p_{ij} = p_{ii}/\tau = \frac{\sigma}{\sigma-1}\frac{c}{\tau}$ to substitute for the prices, we can rewrite Q_i:

$$Q_i = \left(\frac{(n_i + m)\left[\left(\frac{\sigma}{\sigma-1}c\right)^{-\sigma}\right]^{\pi} + n_j\left[\left(\frac{\sigma}{\sigma-1}\frac{c}{\tau}\right)^{-\sigma}\right]^{\pi}}{\left[(n_i + m)\left(\frac{\sigma}{\sigma-1}c\right)^{1-\sigma} + n_j\left(\frac{\sigma}{\sigma-1}\frac{c}{\tau}\right)^{1-\sigma}\right]^{\pi}} (\theta L_i)^{\pi} \right)^{\frac{1}{\pi}} \tag{C.12}$$

Simplifying (C.12):

$$Q_i = \left((n_i + m) + n_j \tau^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \times \theta L_i \times \frac{\sigma - 1}{\sigma c} \tag{C.13}$$

Writing $Y = (1-\theta)L_i$ and substituting for Y and Q_i in the utility function, we can write the the utility function of a representative individual in country i in the post-PTA case as follows:

$$U_i^{Post} = C \times \left(n_i + m + n_j \tau^{\sigma-1}\right)^{\frac{\theta}{\sigma-1}} \tag{C.14}$$

where $C = \theta^{\theta}(1-\theta)^{1-\theta}\left(\frac{\sigma-1}{\sigma c}\right)^{\theta}$ and $n_i, m, n_j \geq 0$