Econ 3790: Business and Economic Statistics

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Chapter 11
Inferences About Population Variances

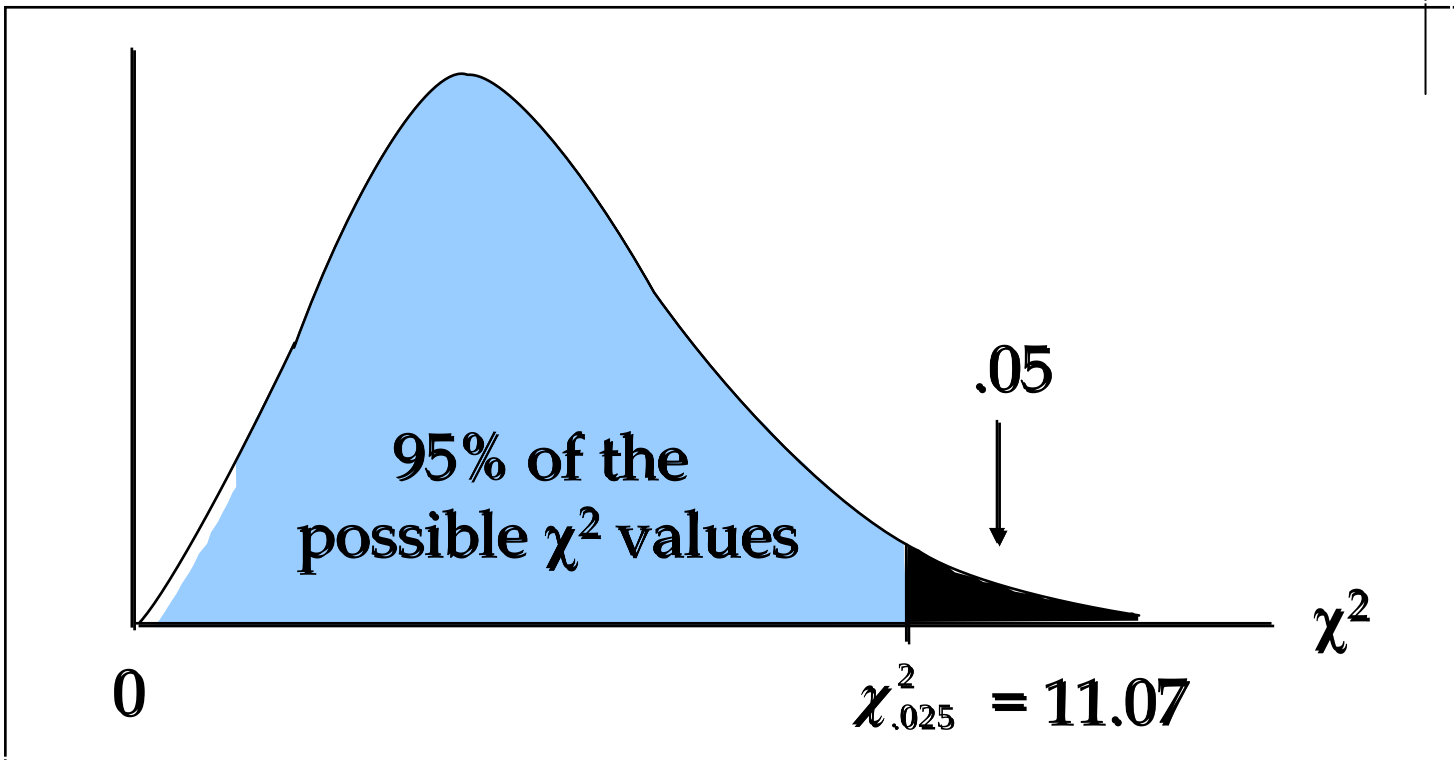
- Inference about a Population Variance
  - Chi-Square Distribution
  - Interval Estimation of σ^2
  - Hypothesis Testing

Chi-Square Distribution

- We will use the notation χ^2_α to denote the value for the chi-square distribution that provides an area of α to the right of the stated χ^2_α value.

- For example, Chi-squared value with 5 degrees of freedom (df) at α =0.05 is 11.07.

Interval Estimation of σ^2



Interval Estimation of σ^2

- Interval Estimate of a Population Variance

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}$$

where the χ^2 values are based on a chi-square distribution with n - 1 degrees of freedom and 1 - α is the confidence coefficient.

Interval Estimation of σ

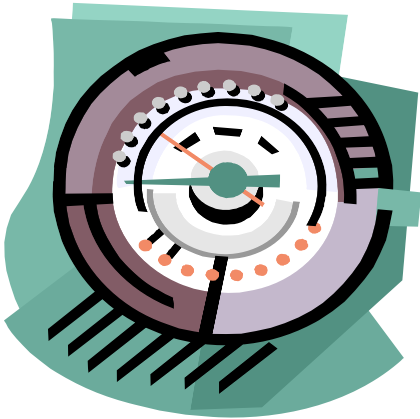
- Interval Estimate of a Population Standard Deviation

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} \le \sigma \le \sqrt{\frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}}$$

Taking the square root of the upper and lower limits of the variance interval provides the confidence interval for the population standard deviation.

Interval Estimation of σ^2

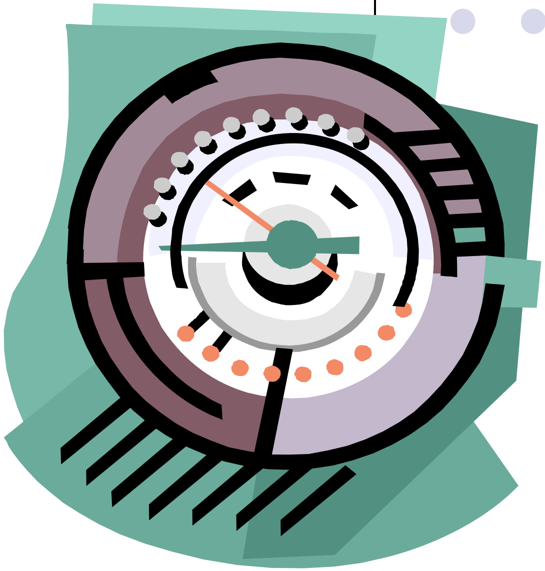
- Example: Buyer's Digest (A): Buyer's Digest rates thermostats manufactured for home temperature control. In a recent test, 10 thermostats manufactured by ThermoRite were selected and placed in a test room that was maintained at a temperature of 68^{o}F. The temperature readings of the ten thermostats are shown on the next slide.



Interval Estimation of σ^2

- Example: Buyer's Digest (A)

  We will use the 10 readings below to develop a 95% confidence interval estimate of the population variance.



Thermostat	1	2	3	4	5	6	7	8	9	10
Temperature	67.4	67.8	68.2	69.3	69.5	67.0	68.1	68.6	67.9	67.2

Interval Estimation of σ^2

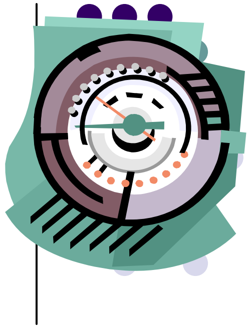
For n - 1 = 10 - 1 = 9 d.f. and α = .05

- Selected Values from the Chi-Square Distribution Table

Degrees of Freedom	Area in Upper Tail .99	.975	.95	.90	.10	.05	.025	.01
5	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.647	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.[illegible]40	4.865	15.987	18.307	20.483	23.209

Our $\chi^2_{.975}$ value

Interval Estimation of σ^2



- Sample variance s^2 provides a point estimate of σ^2.

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{6.3}{9} = .70$$

- A 95% confidence interval for the population variance is given by:

$$\frac{(10-1).70}{19.02} \le \sigma^2 \le \frac{(10-1).70}{2.70}$$

$$.33 \le \sigma^2 \le 2.33$$

Hypothesis Testing about a Population Variance

- Left-Tailed Test
  - Hypotheses

$$H_0: \sigma^2 \geq \sigma_0^{\,2}$$
$$H_a: \sigma^2 < \sigma_0^2$$

where σ_0^2 is the hypothesized value for the population variance

  - Test Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Hypothesis Testing
About a Population Variance

- Left-Tailed Test (continued)
  - Rejection Rule

Critical value approach: Reject H_0 if $\chi^2 \le \chi^2_{(1-\alpha)}$

p-Value approach: Reject H_0 if p-value $\le \alpha$

where $\chi^2_{(1-\alpha)}$ is based on a chi-square distribution with n - 1 d.f.

Hypothesis Testing About a Population Variance

- Right-Tailed Test
  - Hypotheses

$$H_0: \sigma^2 \le \sigma_0^2$$
$$H_a: \sigma^2 > \sigma_0^2$$

where σ_0^2 is the hypothesized value for the population variance

  - Test Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Hypothesis Testing About a Population Variance

- Right-Tailed Test (continued)
  - Rejection Rule

Critical value approach: Reject H_0 if $\chi^2 \geq \chi^2_\alpha$

p-Value approach: Reject H_0 if p-value $\leq \alpha$

where χ^2_α is based on a chi-square distribution with n - 1 d.f.

Hypothesis Testing
About a Population Variance

- Two-Tailed Test
  - Hypotheses

$$H_0\colon \sigma^2 = \sigma_0^2$$
$$H_a\colon \sigma^2 \neq \sigma_0^2$$

where σ_0^2 is the hypothesized value for the population variance

  - Test Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Hypothesis Testing
About a Population Variance

- Two-Tailed Test (continued)
  - Rejection Rule

Critical value approach:

Reject H_0 if $\chi^2 \le \chi^2_{(1-\alpha/2)}$ or $\chi^2 \ge \chi^2_{\alpha/2}$

p-Value approach:

Reject H_0 if *p*-value $\le \alpha$

where $\chi^2_{(1-\alpha/2)}$ and $\chi^2_{\alpha/2}$ are based on a chi-square distribution with n - 1 d.f.

Hypothesis Testing
About a Population Variance

Example: Buyer's Digest (B): Recall that Buyer's Digest is rating ThermoRite thermostats. Buyer's Digest gives an "acceptable" rating to a thermostat with a temperature variance of 0.5 or less.

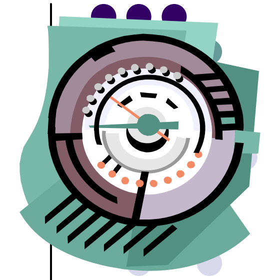
We will conduct a hypothesis test (with $\alpha = .10$) to determine whether the ThermoRite thermostat's temperature variance is "acceptable".

Hypothesis Testing
About a Population Variance

- Example: Buyer's Digest (B)
  - Using the 10 readings, we will conduct a hypothesis test (with α = .10) to determine whether the ThermoRite thermostat's temperature variance is "acceptable".

Thermostat	1	2	3	4	5	6	7	8	9	10
Temperature	67.4	67.8	68.2	69.3	69.5	67.0	68.1	68.6	67.9	67.2

Hypothesis Testing
About a Population Variance



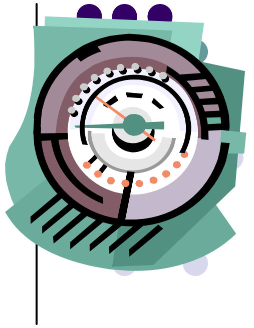
- Hypotheses

$$H_0: \sigma^2 \le 0.5$$
$$H_a: \sigma^2 > 0.5$$

- Rejection Rule

Reject H_0 if $\chi^2 \ge 14.684$

Hypothesis Testing About a Population Variance



For n - 1 = 10 - 1 = 9 d.f. and α = .10

Selected Values from the Chi-Square Distribution Table

Degrees of Freedom	Area in Upper Tail .99	.975	.95	.90	.10	.05	.025	.01
5	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
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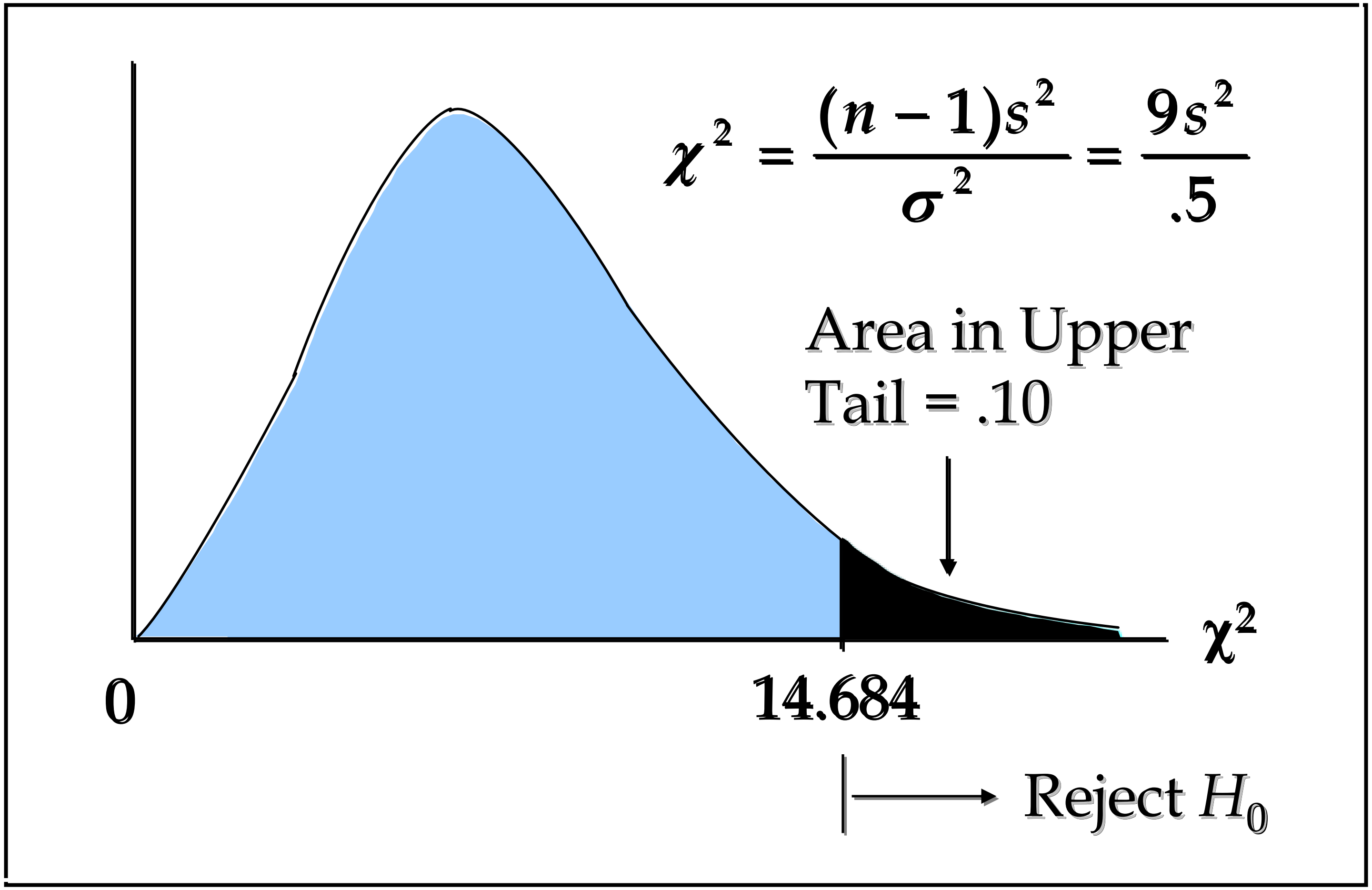
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209

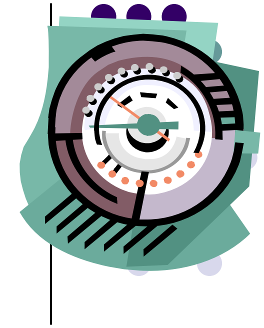
Our $\chi^2_{.10}$ value

Hypothesis Testing
About a Population Variance

- Rejection Region



Hypothesis Testing
About a Population Variance



- The sample variance $s^2 = 0.7$

- Test Statistic

$$\chi^2 = \frac{9(.7)}{.5} = 12.6$$

- Conclusion

Because $\chi^2 = 12.6$ is less than 14.684, we cannot reject H_0. The sample variance $s^2 = .7$ is insufficient evidence to conclude that the temperature variance for ThermoRite thermostats is unacceptable.