Econ 3790: Business and Economics Statistics

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Chapter 4
Introduction to Probability

- Some basic definitions and relationships of probability

Some Definitions

- **Experiment**: A process that generates well-defined outcomes. For example, Tossing a coin, Rolling a die or Playing Blackjack

- **Sample Space**: is the set for all experimental Outcomes. For example, sample space for an experiment of tossing a coin is:

$$S=\{Head, Tail\}$$

Or rolling a die is:

$$S=\{1, 2, 3, 4, 5, 6\}$$

Definitions (Cont'd)

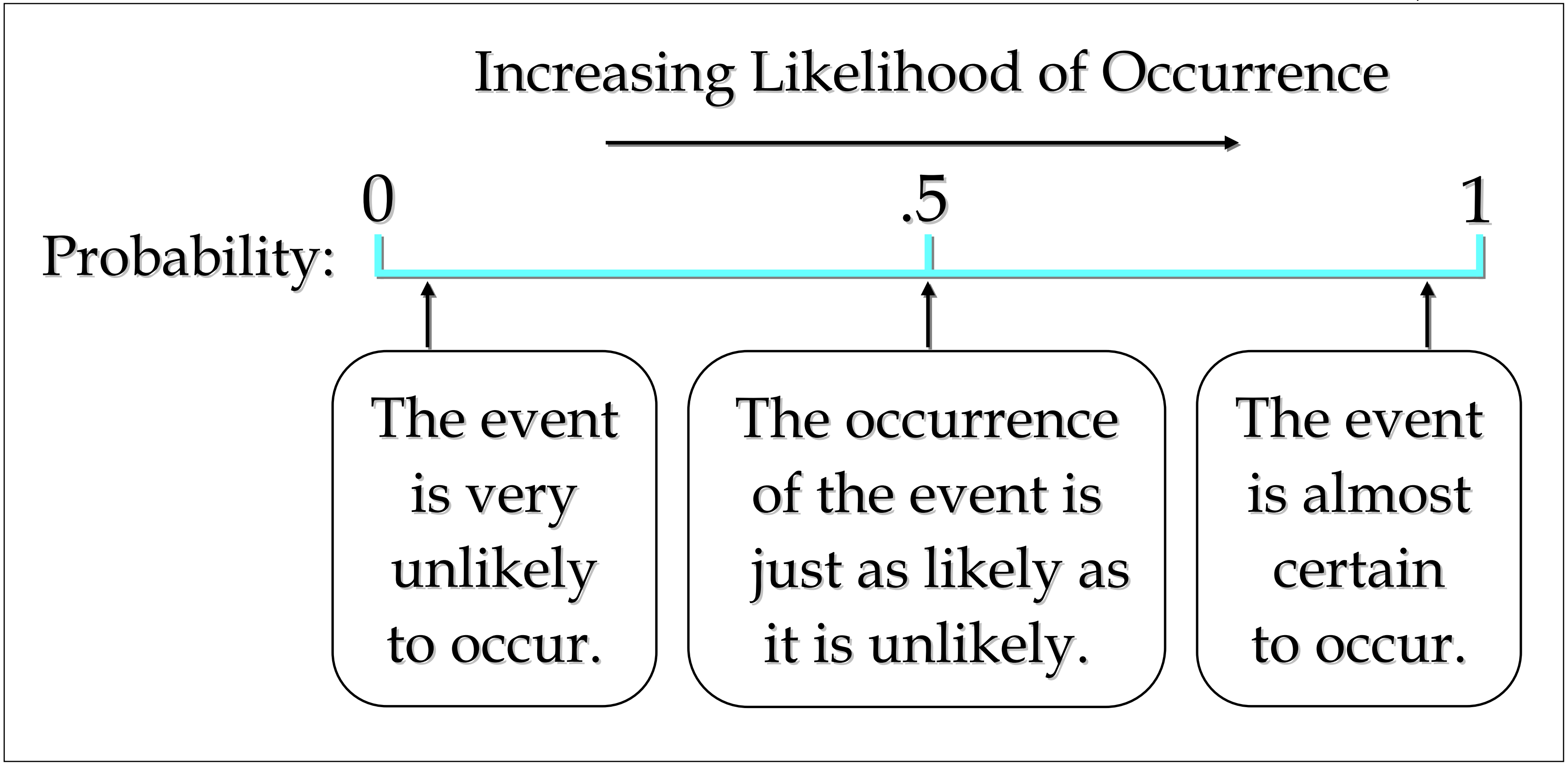
- **<u>Event</u>**: a collection of outcomes or sample points. For example, if our experiment is rolling a die, we can call an incidence of getting a number greater than 3 an event 'A'.

Basic Rules of Probability

1. Probability of any outcome can never be negative or greater than 1.

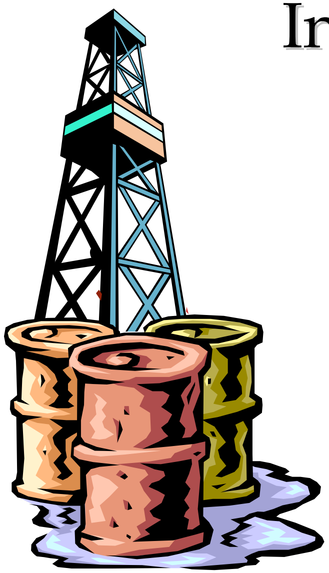
2. The sum of the probabilities of all the possible outcomes of an experiment is 1.

Probability as a Numerical Measure of the Likelihood of Occurrence



Example: Bradley Investments

- Bradley has invested in a stock named Markley Oil. Bradley has determined that the possible outcomes of his investment three months from now are as follows.



Investment Gain or Loss (in $000)
10
5
0
–20

Example: Bradley Investments

- Experiment: Investing in stocks
- Sample Space: $S = \{10, 5, 0, -20\}$
- Event: Making a positive profit (Lets call it 'A')

  $A = \{10, 5\}$

  What is the event for not making a loss?

Assigning Probabilities

- **Classical Method**

  Assigning probabilities based on the assumption of equally likely outcomes

- **Relative Frequency Method**

  Assigning probabilities based on experimentation or historical data

- **Subjective Method**

  Assigning probabilities based on judgment

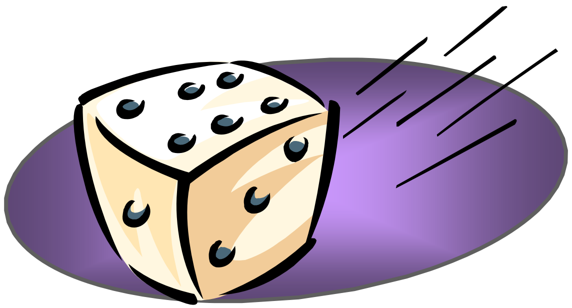
Classical Method

- Assigning probabilities based on the assumption of <u>equally likely outcomes</u>

- If an experiment has *n* possible outcomes, this method would assign a probability of 1/*n* to each outcome.

Example

- Experiment: Rolling a die
- Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$
- Probabilities: Each sample point has a 1/6 chance of occurring



Example

- Experiment: Tossing a Coin
- Sample Space: S = {H, T}
- Probabilities: Each sample point has 1/2 a chance of occurring

Relative Frequency Method

- Assigning probabilities based on experimentation or historical data

Example: Lucas Tool Rental

- Lucas Tool Rental would like to assign probabilities to the number of car polishers it rents each day. Office records show the following frequencies of daily rentals for the last 40 days.

Relative Frequency Method

- Example: Lucas Tool Rental

Number of Polishers Rented	Number of Days
0	4
1	6
2	18
3	10
4	2

Relative Frequency Method

- Each probability assignment is given by dividing the frequency (number of days) by the total frequency (total number of days).

Number of Polishers Rented	Number of Days	Probability
0	4	.10
1	6	.15
2	18	.45
3	10	.25
4	2	.05
	40	1.00

4/40

Example: Favorite Party

Party	Value	Votes	Relative Fre.
Rep	1	5	0.24
Dem	2	14	0.67
Greens	3	0	0.0
None	4	2	0.09
		21	1.00

Subjective Method

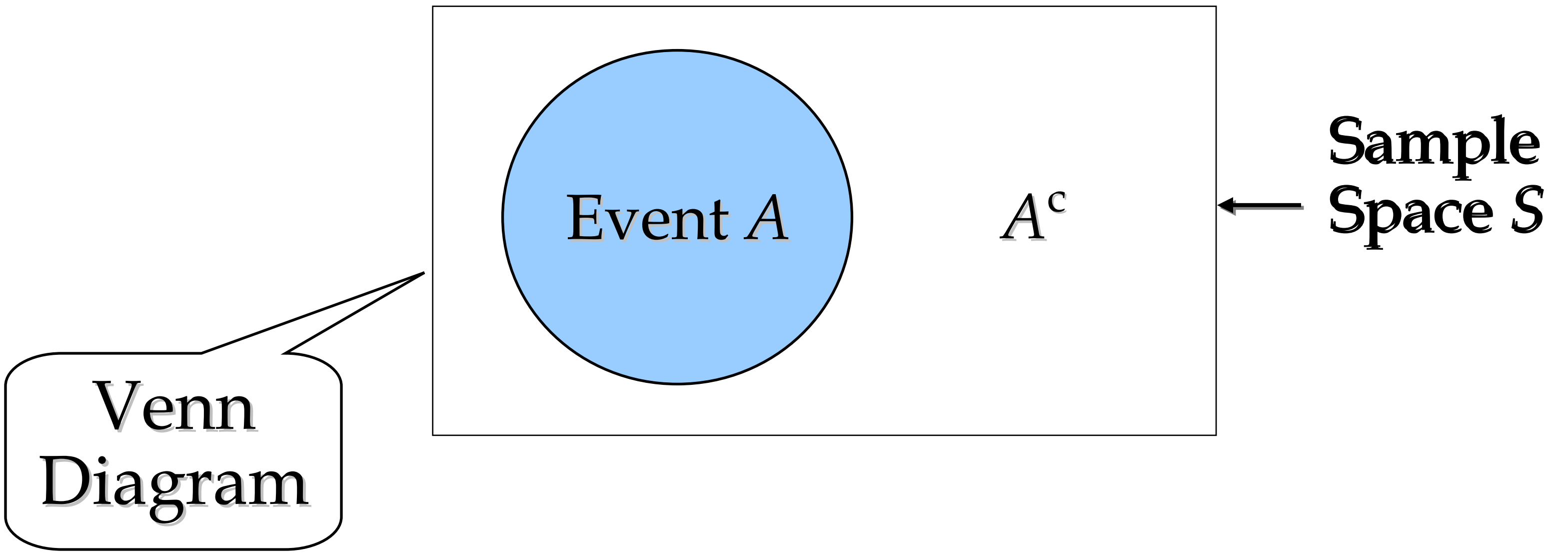
- When economic conditions and a company's circumstances change rapidly it might be inappropriate to assign probabilities based solely on historical data.
- We can use any data available as well as our experience and intuition, but ultimately a probability value should express our degree of belief that the experimental outcome will occur.
- The best probability estimates often are obtained by combining the estimates from the classical or relative frequency approach with the subjective estimate.

Some Basic Relationships of Probability

- Complement of an Event
- Union of Two Events
- Intersection of Two Events
- Mutually Exclusive Events

Complement of an Event

- Complement of an event A is the event consisting of all outcomes or sample points that are not in A and is denoted by A^c.



Example: Rolling a die

- Event A: Getting a number greater than or equal to 3

$$A = \{3, 4, 5, 6\}$$

$$A^c = \{1, 2\}$$

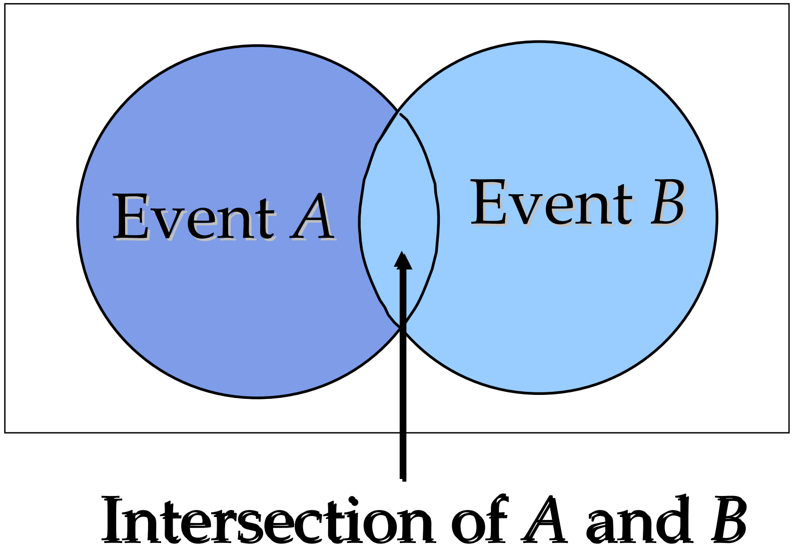
- Event B: Getting a number greater than 1, but less than 5

$$B = \{???\}$$

$$B^c = \{???\}$$

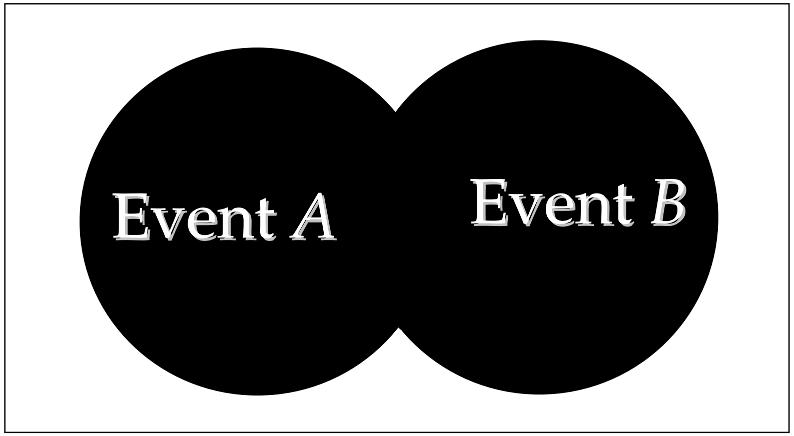
Intersection of two events

- The intersection two events A and B is an event consisting of all sample points that are both in A and B, and is denoted by A ∩ B.



Union of two events

- The Union two events A and B is an event consisting of all sample points that are in A or B or both A and B, and is denoted by A U B.



Union of *A* and *B*

Example: Rolling a Die (Cont'd)

$$A \cap B = \{3, 4\}$$

$$A \cup B = \{2, 3, 4, 5, 6\}$$

Lets find the following probabilities:

P(A) = Outcomes of A / Total Number of Outcomes

$= 4/6 = 2/3$

$P(B) = ?$

$P(A \cap B) = ?$

$P(A \cup B) = ?$

Addition Law

- According to the Addition law, the probability of the event A or B or both can also be written as

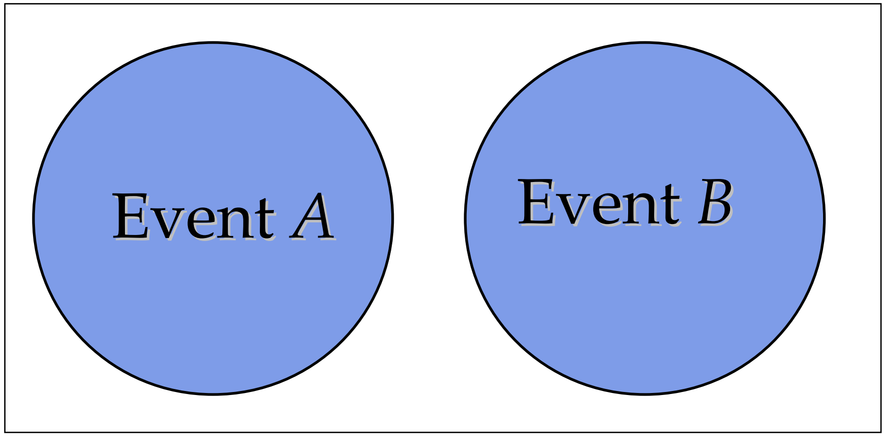
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- In our rolling the die example,

$$P(A\ U\ B) = 2/3 + 1/2 - 1/3 = 5/6$$

Mutually Exclusive Events

- Two events are said to be Mutually Exclusive if, when one event occurs, the other can not occur.
- Or if they do not have any common sample points.



Mutually Exclusive Events

- When Events A and B are mutually exclusive, $P(A \cap B) = 0$.
- The Addition Law for mutually exclusive events is

$$P(A \cup B) = P(A) + P(B)$$

there's no need to include "$- P(A \cap B)$"

Example: Mutually Exclusive Events

- Suppose C is an event of getting a number less than 3 on one roll of a die.

$$C = \{1, 2\}$$

$$A = \{3, 4, 5, 6\}$$

$$P(A \cap C) = 0$$

- Events A and C are mutually exclusive.

Conditional Probability

- The probability of an event (Lets say A) given that another event (Lets say B) has occurred is called Conditional Probability of A.
- It is denoted by P(A | B).

- It can be computed using the following formula:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Rolling the Die Example

- $P(A \cap B) = 1/3$
  - $P(A) = 2/3$
  - $P(B) = 1/2$

- $P(A \mid B) = P(A \cap B) / P(B) = (1/3)/(1/2) = 2/3$

- $P(B \mid A) = P(A \cap B) / P(A) = (1/3)/(2/3) = 1/2$

Multiplication Law

- The multiplication law provides the way to calculate the probability of intersection of two events and is written as follows:

$$P(A \cap B) = P(B) \times P(A \mid B)$$

Independent Events

- If the probability of an event A is not changed or affected by the existence of another event B, then A and B are independent events.
- A and B are independent iff

$$P(A \mid B) = P(A)$$

OR

$$P(B \mid A) = P(B)$$

Multiplication Law for Independent Events

- In case of independent events, the Multiplication Law is written as

$$P(A \cap B) = P(A)P(B)$$

Rolling the Die Example

- So there are two ways of checking whether two events are independent or not:

1. Conditional Probability Method:

$$P(A \mid B) = 2/3 = P(A)$$

$$P(B \mid A) = 1/2 = P(B)$$

- A and B are independent.

Rolling the Die Example

2. The second way is using the Multiplication Law for independent events.

$$P(A \cap B) = 1/3$$

$$P(A) = 2/3$$

$$P(B) = 1/2$$

$$P(A).P(B) = 1/3$$

- Since $P(A \cap B) = P(A).\ P(B)$, A and B are independent events.

Education and Income Data

Highest Grade Completed	Annual Income			
	<$25k	$25k-50k	>$50k	Total
Not HS Grad	19638	4949	1048	25635
HS Grad	34785	25924	10721	71430
Bachelor’s	10081	13680	17458	41219
Total	64504	44553	29227	138284

Education and Income Data

- There are two experiments here:

1. Highest Grade Completed.

   S_1 = {not HS grad, HS grad, Bachelor's}

2. Annual Income.

   S_2={<$25K, $25K-50K, >50K}

- What does each cell represent in the above crosstab?

Education and Income Data

Highest Grade Completed	Annual Income			
	<$25k	$25k-50k	>$50k	Total
Not HS Grad	19638/ 138284 =0.14	4949/ 138284 =0.04	1048/ 138284 =0.01	25635/ 138284 =0.19
HS Grad	34785/ 138284 =0.25	25924/ 138284 =0.19	10721/ 138284 =0.08	71430/ 138284 =0.52
Bachelor’s	10081/ 138284 =0.07	13680/ 138284 =0.10	17458/ 138284 =0.13	41219/ 138284 =0.30
Total	64504/ 138284 =0.47	44553/ 138284 =0.32	29227/ 138284 =0.21	138284/ 138284 =1.00

Education and Income Data

- P(Bachelor’s) = P(Bachelor’s and <25K)
  + P(Bachelor’s and 25-50K)
  + P(Bachelor’s and >50K)
  = 0.07+0.10+0.13 =0.30

- P(>$50K) = P(Not HS and >50K)
  + P(HS grad and >50K)
  + P(Bachelor’s and >50K)
  = 0.01+0.08+0.13 = 0.21

Education and Income Data

- Lets define an event A as the event of making >$50K.

$$A=\{>\$50K\}$$

$$P(A) = 0.21$$

- Lets define another even B as the event of having a HS degree.

$$B= \{HS\ Grad\}$$

$$P(B) = 0.52$$

Rules of Probability

- A and B is an event of having an income >$50K and being a HS graduate:

  P(A and B) = 0.08

- A or B is an event of having an income >$50K or being a HS graduate or both:

  P(A or B) = P(A) + P(B) – P(A and B)

  = 0.21 + 0.52 – 0.08 = 0.65

Education and Income Data

- Event of making >$50K given the event of being a HS graduate:

$$P(A \mid B) = P(A \text{ and } B) / P(B)$$
$$= 0.08 / 0.52 = 0.15$$

- Are A and B independent?

1. $P(A \mid B) = 0.15 \neq P(A) = 0.21$
2. $P(A \text{ and } B) = 0.08 \neq P(A)*P(B) = 0.21*0.52 = 0.11$

$\rightarrow$ A and B are not independent.

- Are Annual Income and Highest Grade Completed independent?

Education and Income Data

- The probability of any event is the sum of probabilities of its sample points.
- E.g. Lets define an event C as the event of having at least a HS degree.

C= {HS Grad, Bachelor's}

P(C) = P(HS Grad) + P(Bachelor's)

=0.52 + 0.30 =0.82