Econ 3790: Business and Economics Statistics

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Chapter 5

- Random Variables
- Probability Distributions
  - Discrete Distributions
    - Discrete Uniform Probability Distribution
    - Binomial Probability Distribution
  - Continuous Distribution
    - Normal Distribution

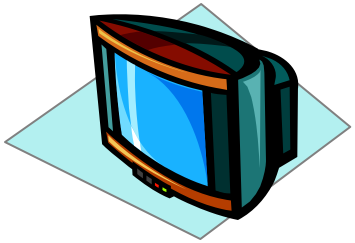
Random Variables

- A random variable is a numerical description of the outcome of an experiment.
- A discrete random variable may assume either a finite number of values or an infinite sequence of values.
- A continuous random variable may assume any numerical value in an interval or collection of intervals.

Example: JSL Appliances

- Discrete random variable with a finite number of values

Let x = number of TVs sold at the store in one day,
where x can take on 5 values (0, 1, 2, 3, 4)

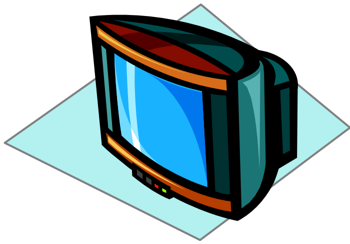


Example: JSL Appliances

- Discrete random variable with an infinite sequence of values

> Let x = number of customers arriving in one day,
> where x can take on the values 0, 1, 2, . . .

We can count the customers arriving, but there is no finite upper limit on the number that might arrive.



Random Variables

Question	Random Variable x	Type
Family size	x = Number of dependents reported on tax return	Discrete
Distance from home to store	x = Distance in miles from home to the store site	Continuous
Own dog or cat	x = 1 if own no pet; = 2 if own dog(s) only; = 3 if own cat(s) only; = 4 if own dog(s) and cat(s)	Discrete

Discrete Probability Distributions

- The probability distribution for a random variable describes how probabilities are distributed over the values of the random variable.

- We can describe a discrete probability distribution with a table, graph, or equation.

Discrete Probability Distributions

- The probability distribution is defined by a probability function, denoted by $p(x)$, which provides the probability for each value of the random variable.

- The required conditions for a discrete probability function are:

$$p(x) \geq 0$$

$$\Sigma p(x) = 1$$

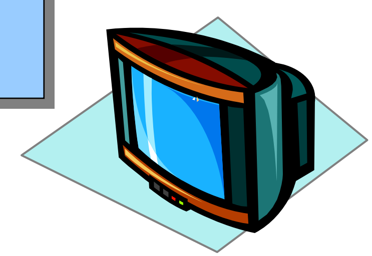
Discrete Probability Distributions

- Using past data on TV sales, …
- a tabular representation of the probability distribution for TV sales was developed.

Units Sold	Number of Days
0	80
1	50
2	40
3	10
4	20
	200

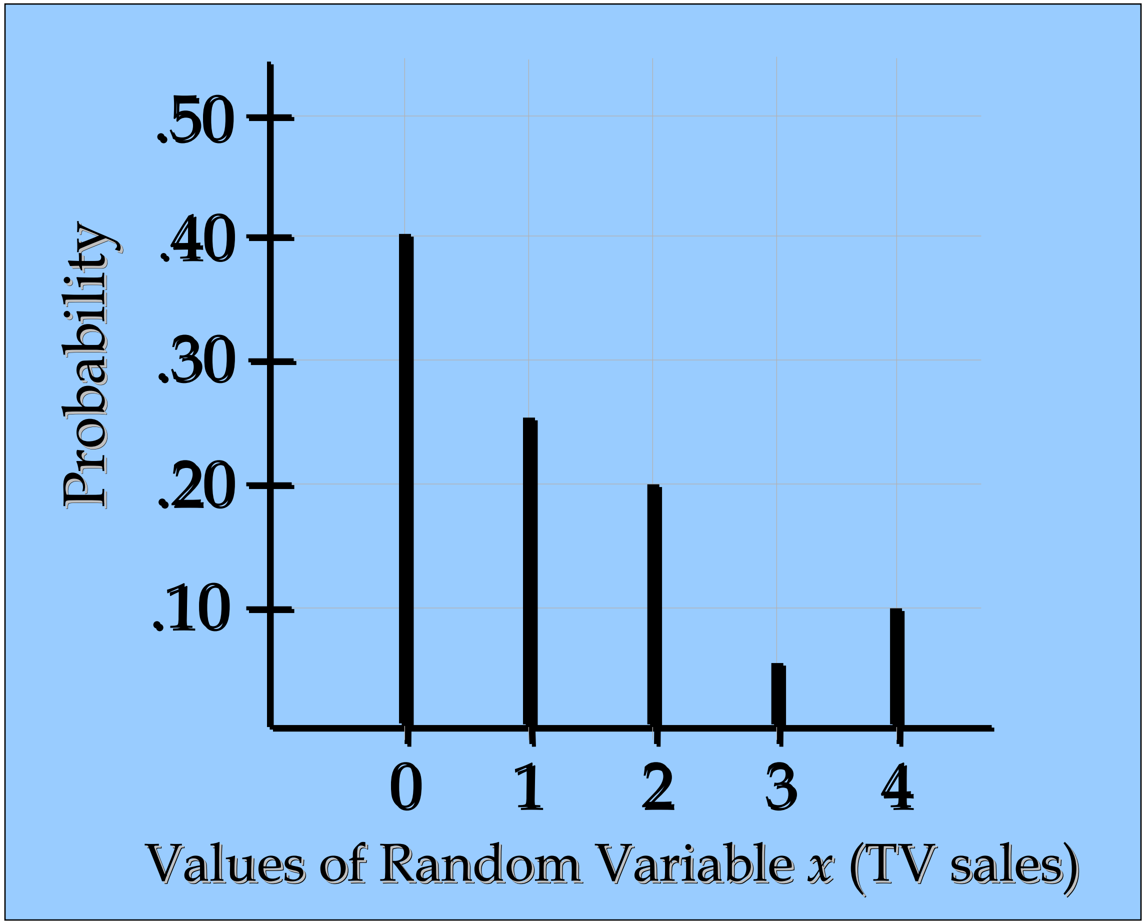
x	$p(x)$
0	.40
1	.25
2	.20
3	.05
4	.10
	1.00

80/200



Discrete Probability Distributions

- Graphical Representation of Probability Distribution



Expected Value and Variance

- The expected value, or mean, of a random variable is a measure of its central location.

$$E(x) = \mu = \Sigma\, p(x) * x$$

- The variance summarizes the variability in the values of a random variable.

$$\text{Var}(x) = \sigma^2 = \Sigma p(x) * (x - \mu)^2$$

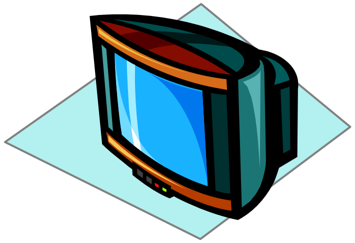
- The standard deviation, σ, is defined as the positive square root of the variance.

Expected Value

x	$p(x)$	$x*p(x)$
0	.40	.00
1	.25	.25
2	.20	.40
3	.05	.15
4	.10	.40

$E(x) = 1.20$

expected number of TVs sold in a day

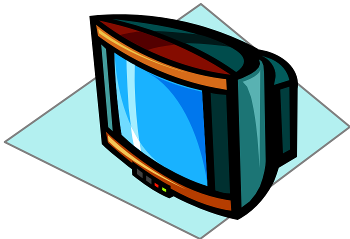


Variance and Standard Deviation

x	$x - \mu$	$(x - \mu)^2$	$p(x)$	$p(x)*(x - \mu)^2$
0	-1.2	1.44	.40	.576
1	-0.2	0.04	.25	.010
2	0.8	0.64	.20	.128
3	1.8	3.24	.05	.162
4	2.8	7.84	.10	.784

Variance of daily sales = σ^2 = 1.660

Standard deviation of daily sales = 1.2884 TVs



Types of Discrete Probability Distributions:

- Uniform
- Binomial

Discrete Uniform Probability Distribution

- The discrete uniform probability distribution is the simplest example of a discrete probability distribution given by a formula.

- The discrete uniform probability function is

$$p(x) = 1/n$$

the values of the random variable are equally likely

where:

n = the number of values the random variable may assume

Discrete Uniform Probability Distribution

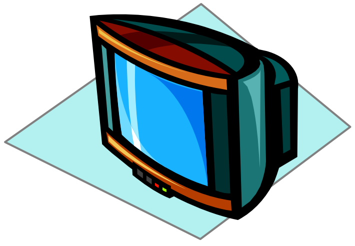
- Suppose, instead of looking at the past sales of the TVs, I assume (or think) that TVs sales have a uniform probability distribution, then the example done above would change as follows:

Expected Value

x	$p(x)$	$x*p(x)$
0	.2	.00
1	.2	.20
2	.2	.40
3	.2	.60
4	.2	.80

$E(x) = 2.0$

expected number of TVs sold in a day

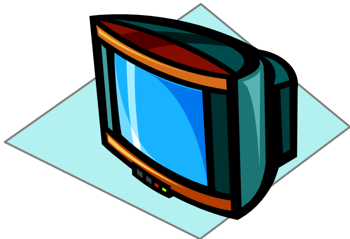


Variance and Standard Deviation

x	$x - \mu$	$(x - \mu)^2$	$p(x)$	$p(x)*(x - \mu)^2$
0	-2.0	4.0	.2	0.8
1	-1.0	1.0	.2	0.2
2	0.0	0.0	.2	0.0
3	1.0	1.0	.2	0.2
4	2.0	4.0	.2	0.8

Variance of daily sales = $\sigma^2 = 2.0$

Standard deviation of daily sales = 1.41 TVs



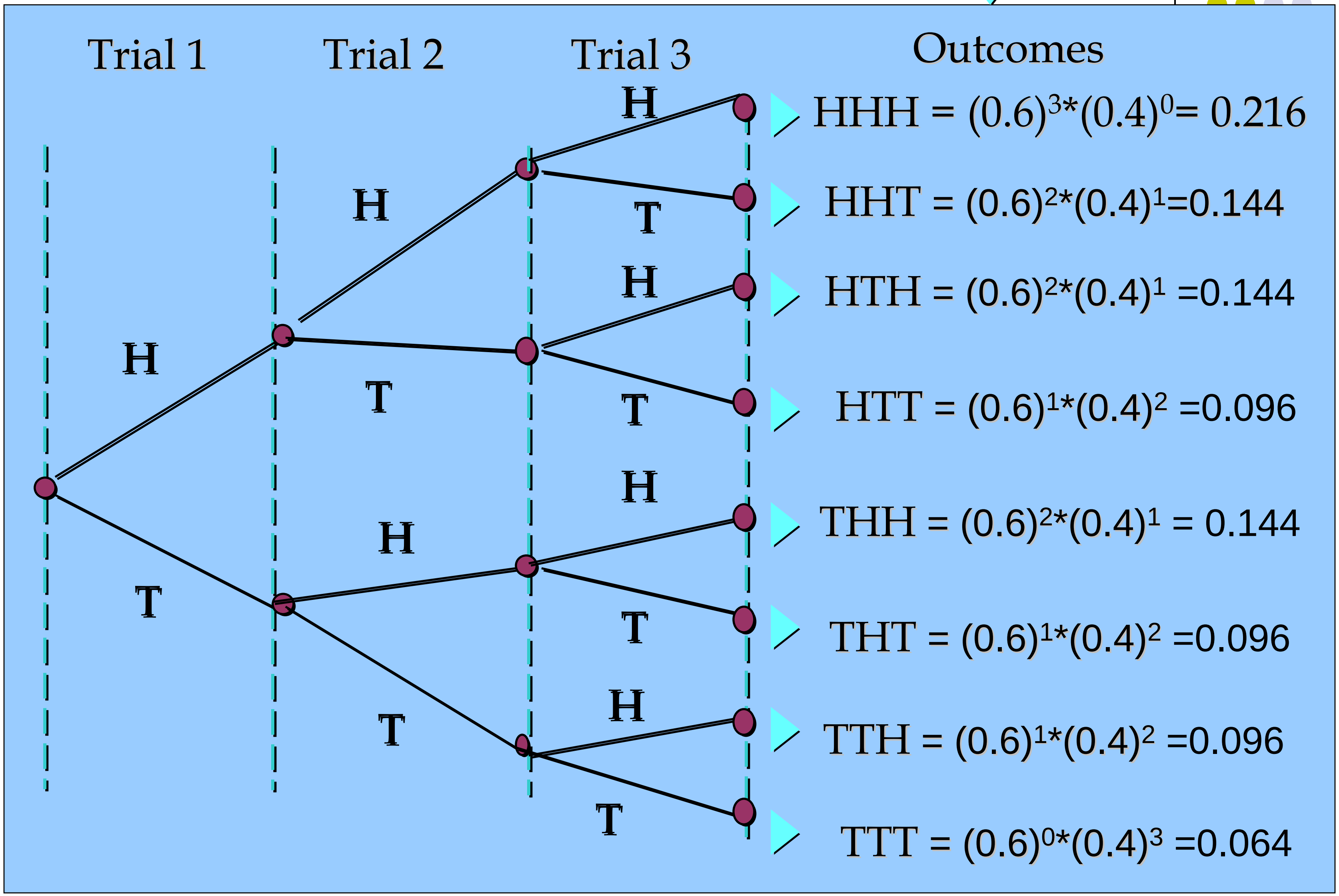
Example: I am bored

- Imagine this situation. There is heavy snowstorm. Everything is shut down. You and everybody in your family have to stay home. You are utterly bored. You catch hold of your sibling and get him or her to play this game.
- The game is to bet on the toss of a coin.

Example: I am bored

- If it turns up heads exactly once in three tosses, you win or otherwise you lose.
- Lets call the event of getting heads on anyone trial as a success. Similarly, the event of getting tails is a failure.
- Suppose the probability of getting heads (or of a success) is 0.6.
- The big question is that you want to find out the probability of getting <u>exactly 1</u> head on three tosses.

Tree Diagram



So, what is the probability of you winning the game?

- What is the random variable here?
- What is the probability of getting 2 heads in three tosses?

$$= P(HHT) + P(HTH) + P(THH)$$

$$= 0.144 + 0.144 + 0.144$$

$$= 0.432$$

Or 43.2%

- How does the probability distribution of our Random variable look?

Binomial Distribution

1. The experiment consists of a sequence of n identical trials.
2. Two outcomes, <u>success</u> and <u>failure</u>, are possible on each trial.
3. The probability of a success, denoted by p, does not change from trial to trial.
4. The trials are independent.

Binomial Distribution

- Our interest is in the number of successes occurring in the n trials.

- We let x denote the number of successes occurring in the n trials.

- Binomial Distribution is highly useful when the number of trials is large.

Binomial Distribution

- Binomial Probability Function

$$\# \ of \ ways \ . \ p^{x} . (1 - p)^{n-x}$$

where:

n = the number of trials

p = the probability of success on any one trial

Counting Rule for Combinations

- Another useful counting rule (esp. when n is large) enables us to count the number of experimental outcomes when *x* objects are to be selected from a set of *N* objects.

- Number of <u>Combinations</u> of *n* Objects Taken *x* at a Time

$$C_x^n = \frac{n!}{x!(n-x)!}$$

where:

$$n! = n(n-1)(n-2)\ldots(2)(1)$$

$$x! = x(x-1)(x-2)\ldots(2)(1)$$

$$0! = 1$$

Example: I am bored

- Using binomial distribution, the probability of 1 head in 3 tosses is

$$= 3.(0.6)^1.(1-0.6)^{3-1}$$

$$= 3.(0.6)^1.(0.4)^2$$

$$= 0.288$$

Example: I am bored

- Suppose, you won. But knowing your sibling, she or he says that bet was getting exactly 2 heads in 3 tosses. Since you are bored, you have no choice but continuing to play:

$$= 3.(0.6)^{1}.(1-0.6)^{3-1}$$

$$= 3.(0.6)^{2}.(0.4)^{1}$$

$$= 0.432$$

Example: I am bored

- She again cheats. She says that bet was getting at least 2 heads in 3 tosses.

- What does this mean: Getting 2 or more heads $\Longrightarrow$ P(2 heads) + P(3 heads)

Example: I am bored

$$P(2\,heads) = 3.(0.6)^2.(1-0.6)^{3-2}$$

$$= 3.(0.6)^2.(0.4)^1$$

$$= 0.432$$

$$P(3\,heads) = 1.(0.6)^3.(1-0.6)^{3-3}$$

$$= 1.(0.6)^3.(0.4)^0$$

$$= 0.216$$

$$P(2\,heads) + P(3\,heads) = 0.432 + 0.216 = 0.648$$

Binomial Distribution

- Expected Value

$$E(x) = \mu = n^*p$$

- Variance

$$\mathrm{Var}(x) = \sigma^2 = np(1-p)$$

- Standard Deviation

$$\sigma = \sqrt{np(1-p)}$$

Example: I am bored

- Mean (or expected value)

$$E(x) = \mu = n*p = 3*0.6 = 1.8$$

- Variance:

$$\text{Var}(x) = \sigma^2 = np(1-p)$$
$$= 3*(0.6)*(1\text{-}0.6) = 0.72$$

- Standard Deviation

$$\sigma = \sqrt{Var(x)} = \sqrt{0.72} = 0.84$$