Econ 3790: Business and Economics Statistics

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Chapter 8: Interval Estimation

- Population Mean: σ Known
- Population Mean: σ Unknown

Margin of Error and the Interval Estimate

- A point estimator cannot be expected to provide the exact value of the population parameter.

- An interval estimate can be computed by adding and subtracting a margin of error to the point estimate.

  Point Estimate +/− Margin of Error

- The purpose of an interval estimate is to provide information about how close the point estimate is to the value of the parameter.

Margin of Error and the Interval Estimate

The general form of an interval estimate of a population mean is

$$\bar{x} \pm \text{Margin of Error}$$

- In order to develop an interval estimate of a population mean, the margin of error must be computed using either:
  - the population standard deviation σ, or
  - the sample standard deviation *s*
- These are also *Confidence Interval*.

Interval Estimate of a Population Mean: σ Known

- Interval Estimate of μ

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

where: $\bar{x}$ is the sample mean

$1 - \alpha$ is the confidence coefficient

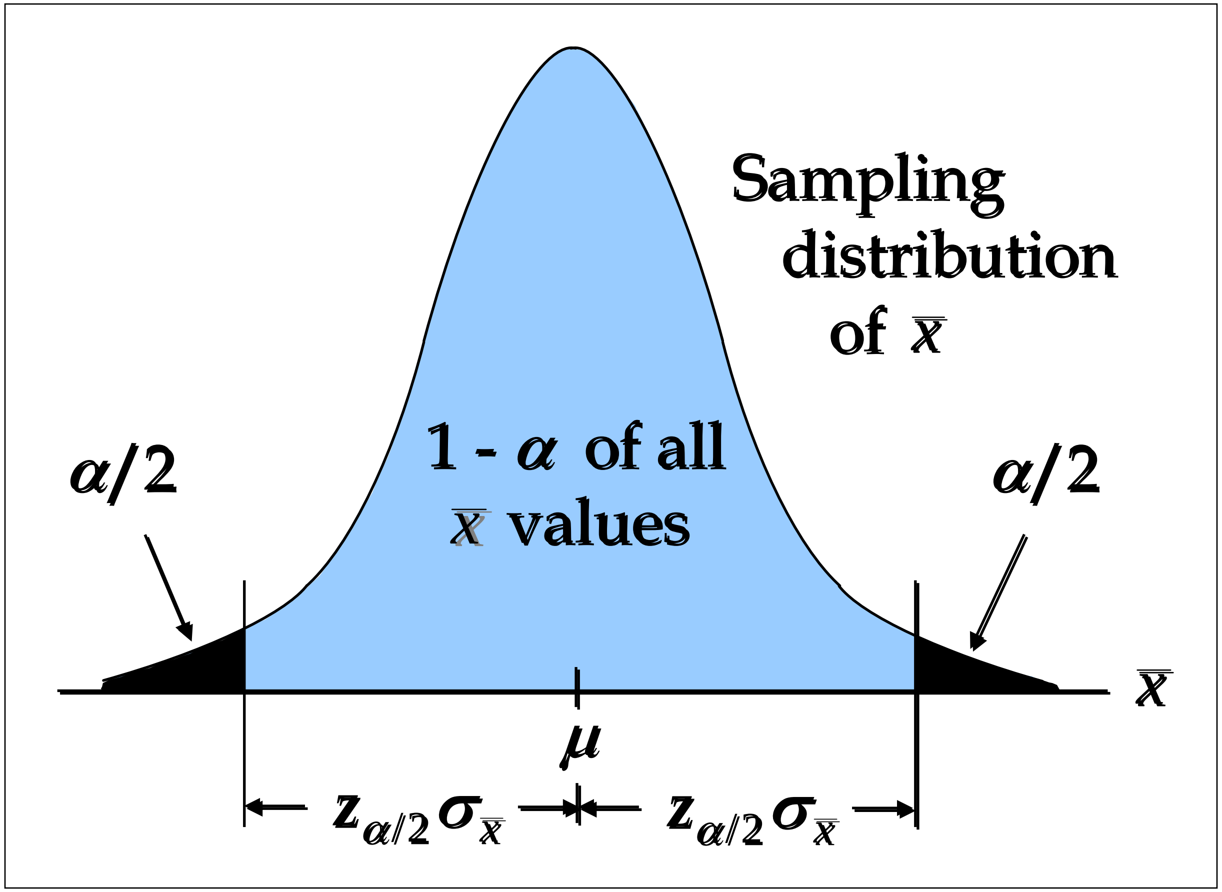
$z_{\alpha/2}$ is the z value providing an area of $\alpha/2$ in the upper tail of the standard normal probability distribution

σ is the population standard deviation

n is the sample size

Interval Estimation of a Population Mean: σ Known

- There is a $1 - \alpha$ probability that the value of a sample mean will provide a margin of error of $z_{\alpha/2}\sigma_{\bar{x}}$ or less.



Summary of Point Estimates Obtained from a Simple Random Sample

Population Parameter	Parameter Value	Point Estimator	Point Estimate
μ = Population mean	40.9	$\bar{x}$ = Sample mean	
σ = Population std. deviation	20.5	s = Sample std. deviation	…….
p = Population proportion	.62	$\bar{p}$ = Sample proportion	

Example: Air Quality

- Consider our air quality example. Suppose the population is *<u>approximately normal</u>* with $\mu = 40.9$ and $\sigma = 20.5$. This is *<u>σ known</u>* case.
- If you guys remember, we picked a sample of size 5 ($n = 5$).
- Given all this information, What is the margin of error at 95% confidence level?

Example: Air Quality

- What is the margin of error at 95% confidence level.

$$z_{0.025}\frac{\sigma}{\sqrt{n}} = 1.96 \times \frac{20.5}{\sqrt{5}} = 1.96 \times 9.2 = 18$$

- We can say with 95% confidence that population mean (μ) is between ± 18 of the sample mean.

$$\bar{x} \pm z_{0.025}\frac{\sigma}{\sqrt{n}}$$

- With 95% confidence, μ is between …. and ……

Interval Estimation of a Population Mean: σ Unknown

- If an estimate of the population standard deviation σ cannot be developed prior to sampling, we use the sample standard deviation *s* to estimate σ.
- This is the σ unknown case.
- In this case, the interval estimate for μ is based on the *t* distribution.
- (We'll assume for now that the population is normally distributed.)

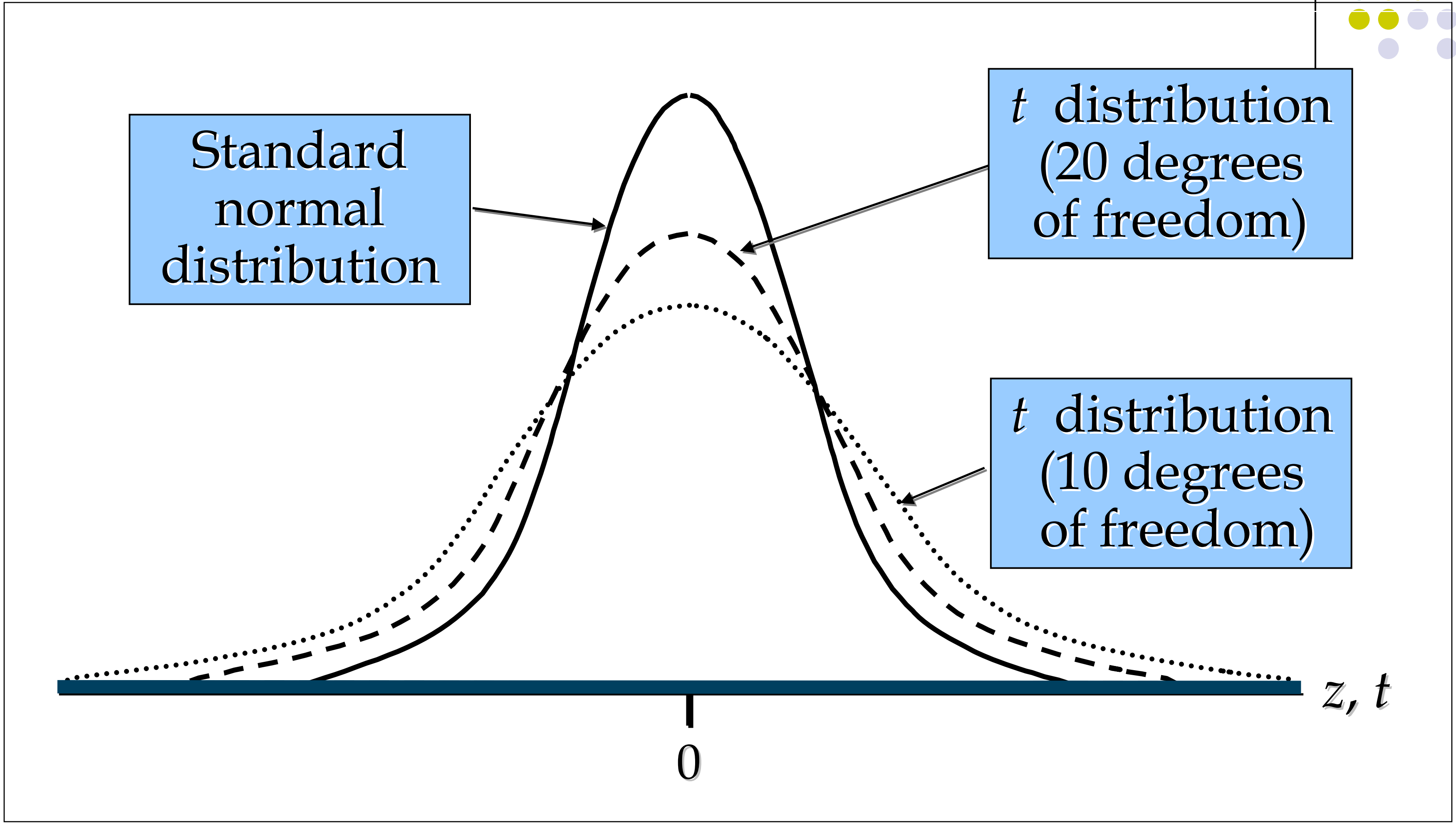
t Distribution

- The t distribution is a family of similar probability distributions.
- A specific t distribution depends on a parameter known as the degrees of freedom.
- Degrees of freedom refer to the number of independent pieces of information that go into the computation of s.

t Distribution

- A *t* distribution with more degrees of freedom has less dispersion.
- As the number of degrees of freedom increases, the difference between the *t* distribution and the standard normal probability distribution becomes smaller and smaller.

t Distribution



t Distribution

- For more than 100 degrees of freedom, the standard normal *z* value provides a good approximation to the *t* value.
- The standard normal *z* values can be found in the infinite degrees (∞) row of the *t* distribution table.

t Distribution

Degrees of Freedom	Area in Upper Tail 0.2	.10	.05	.025	.01	.005
.	.	.	.	.	.	.
50	0.849	1.299	1.676	2.009	2.403	2.678
60	0.848	1.296	1.671	2.000	2.390	2.660
80	0.846	1.292	1.664	1.990	2.374	2.639
100	0.845	1.290	1.660	1.984	2.364	2.626
∞	0.842	1.282	1.645	1.960	2.326	2.576

Standard normal z values

Interval Estimation of a Population Mean: σ Unknown

- Interval Estimate

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where: $1 - \alpha$ = the confidence coefficient

$t_{\alpha/2}$ = the t value providing an area of $\alpha/2$ in the upper tail of a t distribution with *n - 1 degrees of freedom*

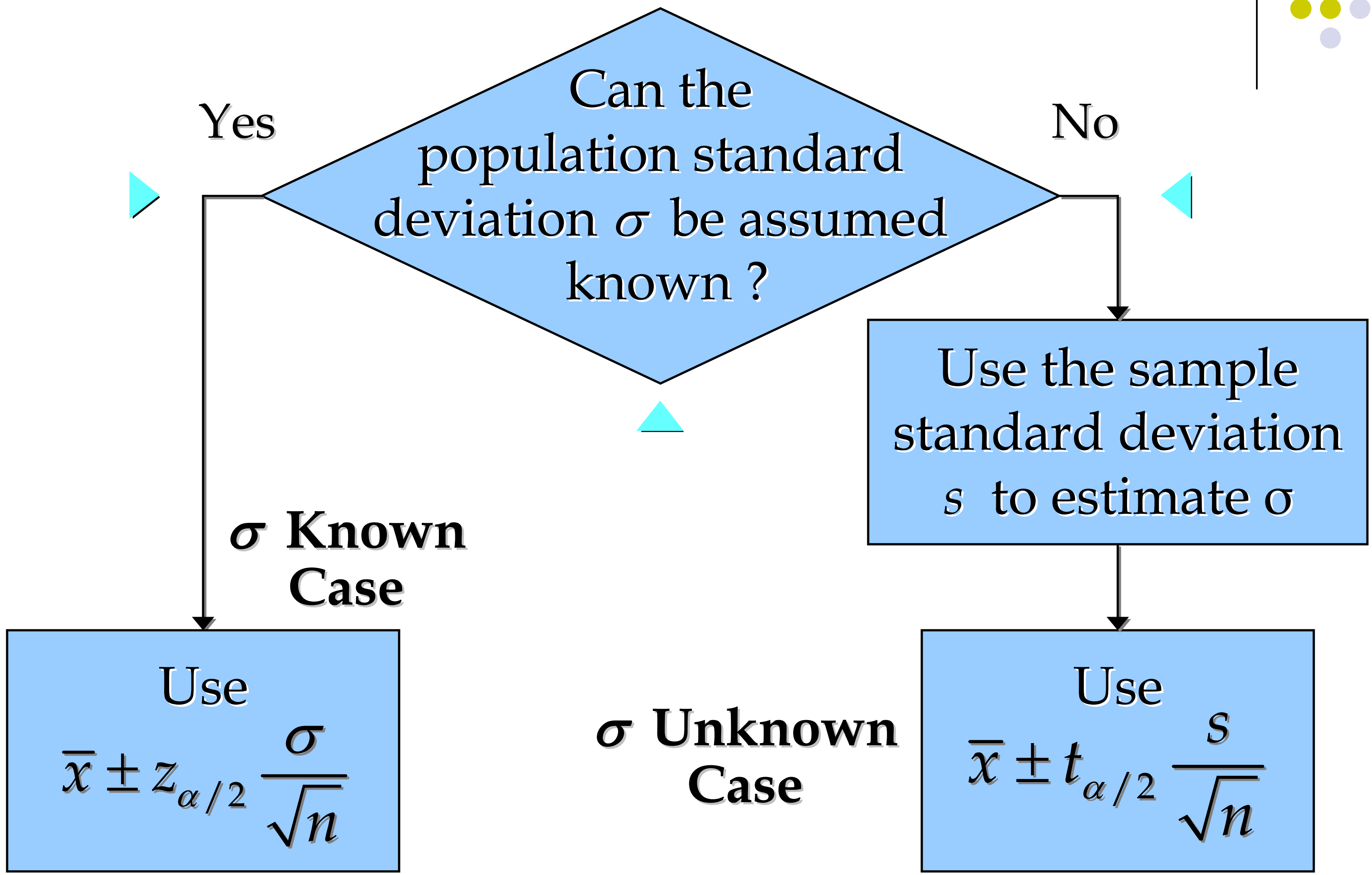
s = the sample standard deviation

Example: Air quality when σ is unknown

- Now suppose that you did not know what σ is. You can estimate using the sample and then use t-distribution to find the margin of error.
- What is 95% confidence interval in this case? The sample size n =5. So, the degrees of freedom for the t-distribution is 4. The level of significance (α) is 0.05. s = ……

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

Summary of Interval Estimation Procedures for a Population Mean



Interval Estimation of a Population Proportion

- The general form of an interval estimate of a population proportion is

$$\bar{p} \pm \text{Margin of Error}$$

Interval Estimation of a Population Proportion

- Interval Estimate

$$\bar{p} \pm z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where: $1-\alpha$ is the confidence coefficient

$z_{\alpha/2}$ is the z value providing an area of $\alpha/2$ in the upper tail of the standard normal probability distribution

$\bar{p}$ is the sample proportion