Econ 3790: Business and Economics Statistics

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Chapter 9, Part A: Hypothesis Tests

- Developing Null and Alternative Hypotheses
- Type I and Type II Errors
- Population Mean: σ Known
- Population Mean: σ Unknown

Developing Null and Alternative Hypotheses

- Hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.
- The null hypothesis, denoted by H_0, is a tentative assumption about a population parameter.
- The alternative hypothesis, denoted by H_a, is the opposite of what is stated in the null hypothesis.
- The alternative hypothesis is what the test is attempting to establish.

Summary of Forms for Null and Alternative Hypotheses about a Population Mean

- The equality part of the hypotheses always appears in the null hypothesis.
- In general, a hypothesis test about the value of a population mean μ must take one of the following three forms (where μ_0 is the hypothesized value of the population mean).

One-tailed (lower-tail)	One-tailed (upper-tail)	Two-tailed
$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$	$H_0: \mu = \mu_0$
$H_a: \mu < \mu_0$	$H_a: \mu > \mu_0$	$H_a: \mu \neq \mu_0$

Type I Error

- A Type I error is rejecting H_0 when it is true.

Type II Error

- A Type II error is accepting H_0 when it is false.
- Statisticians avoid the risk of making a Type II error by using "do not reject H_0" and not "accept H_0".

Type I and Type II Errors

	Population Condition	
Conclusion	**H_0 True** ($\mu \leq 12$)	**H_0 False** ($\mu > 12$)
Accept H_0 (Conclude $\mu \leq 12$)	Correct Decision	Type II Error
Reject H_0 (Conclude $\mu > 12$)	Type I Error	Correct Decision

Some Definitions:

- Level of Significance: The probability of making Type I error.
- Critical Value: The value (determined by the level of significance) that establishes the boundary of the rejection region.
- Test Statistic: A computed value which is compared to the critical value to reject or not reject the null.
- *p*-value: is the probability of getting a value more extreme than the test statistic.

One Sample z-test: Steps of Hypothesis Testing **When σ is known**

Step 1. Develop the null and alternative hypotheses.

Step 2. Specify the level of significance α. This will define the critical value for the test.

Step 3. Compute the value of the test statistic (z) or the p-value corresponding to that test statistic.

Steps of Hypothesis Testing **When σ is known**

Step 4.

- Lower Tailed test $(H_a : \mu < \mu_0)$:

Reject H_0 if $z \le -z_\alpha$ or *p*-value $\le \alpha$.

- Upper Tailed test $(H_a : \mu > \mu_0)$:

Reject H_0 if $z \ge +z_\alpha$ or *p*-value $\le \alpha$.

- Two-Tailed test $(H_a : \mu \ne \mu_0)$:

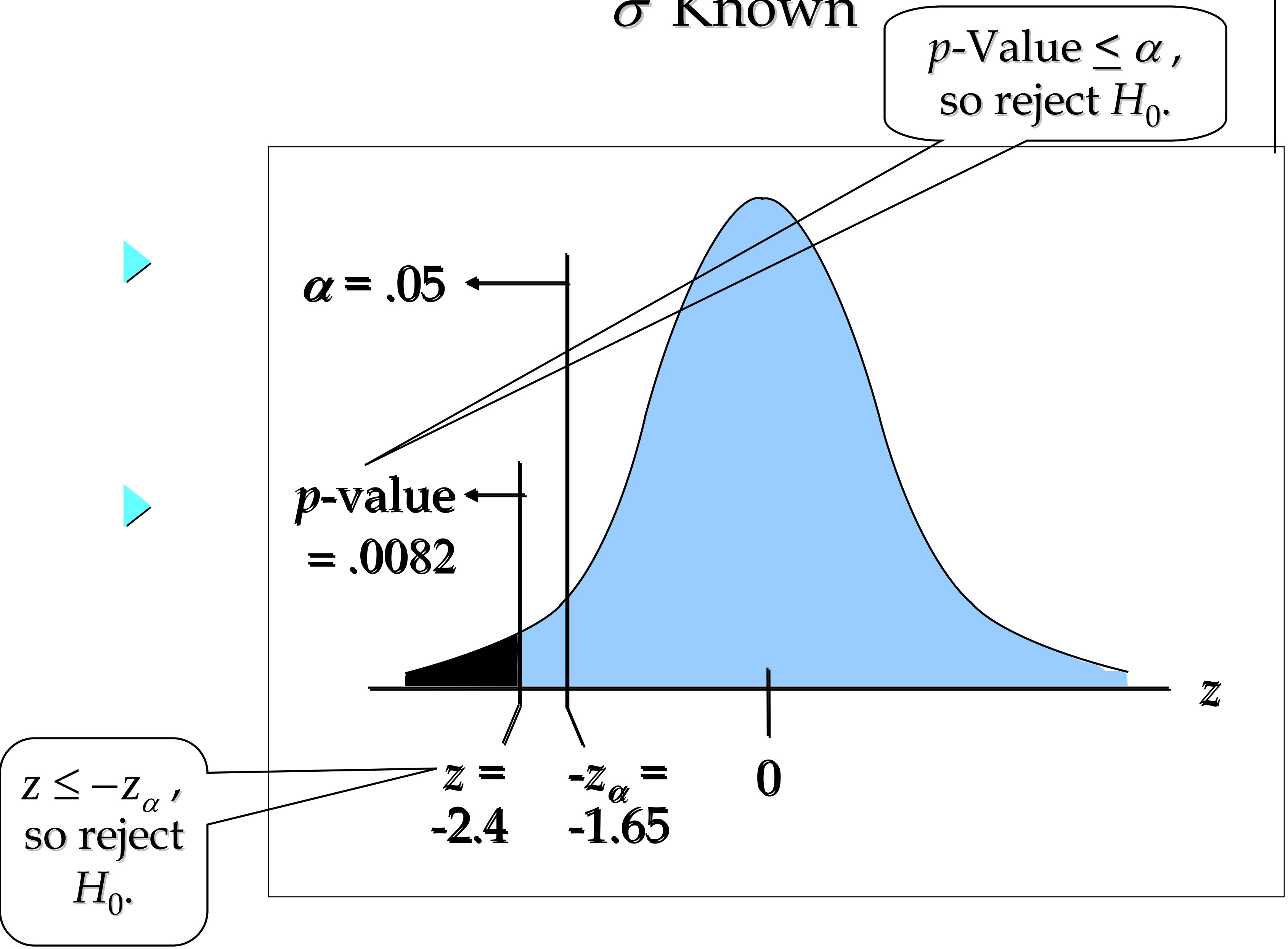
Reject H_0 if $z \le -z_{\alpha/2}$ or $z \ge +z_{\alpha/2}$ or *p*-value $\le \alpha$.

Hypothesis Testing When σ is known

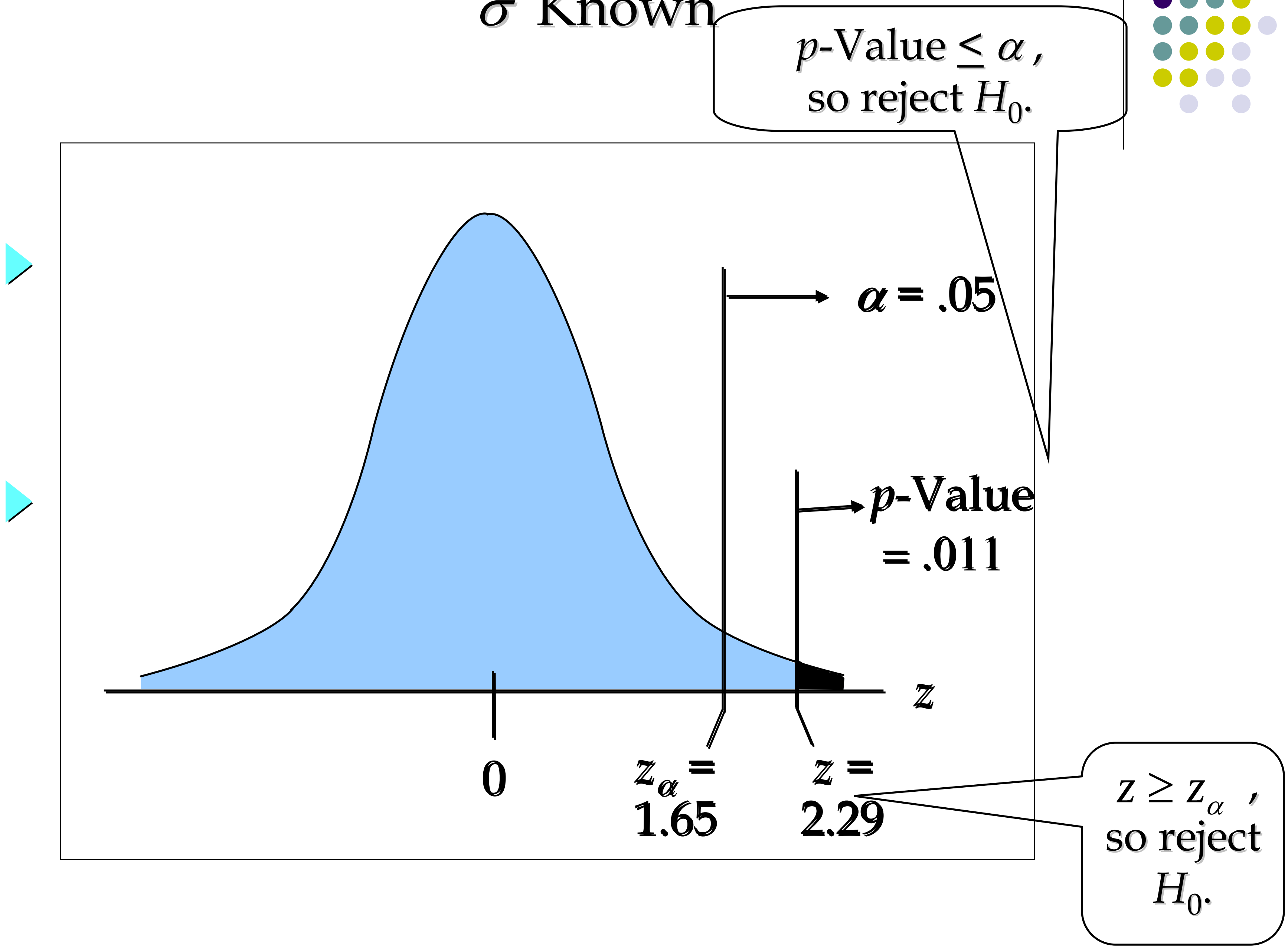
- The test statistic in this case is given by

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

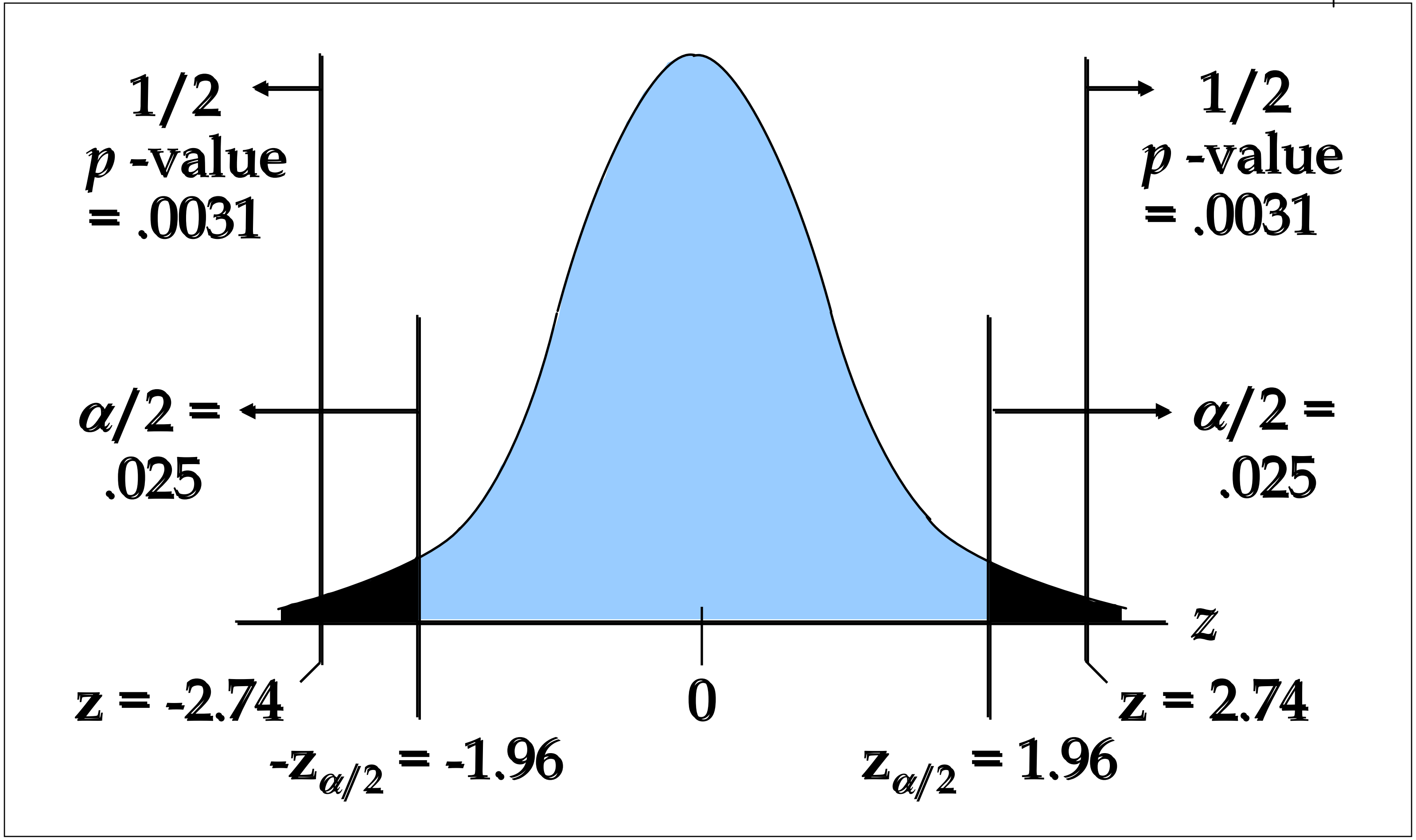
Lower-Tailed Test About a Population Mean: σ Known



Upper-Tailed Test About a Population Mean: σ Known



Two-Tailed Tests About a Population Mean: σ Known



Example of Lower-Tailed Test: Air Quality Data

- Suppose xyz institute claims that air quality is bad in the US. You want to test this claim. Further suppose the level of significance (α) is 5% and a sample of size 5 is selected.
- Step 1:

$$H_0 : \mu \geq 50$$

$$H_a : \mu < 50$$

- Step 2:

The critical value corresponding to α =0.05 is -1.65

Example of Lower-Tailed Test: Air Quality Data

- Step 3: Compute the value of test statistic.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

- Step 4: Make your conclusion using the critical value and p-value approaches.

Example of Two-Tailed Test: Air Quality Data

- Suppose again xyz institute claimed that average air quality (average value of PMI) in the US is 48. Test this claim the 5% level of significance.

- How does sample size affect your conclusions?

Tests About a Population Mean: σ Unknown (One sample t-test)

- Test Statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

This test statistic has a t distribution with n - 1 degrees of freedom.

Tests About a Population Mean: σ Unknown (One sample t-test)

- Rejection Rule
  - H_a: $\mu < \mu_0$ Reject H_0 if $t \leq -t_\alpha$ or p –value $\leq \alpha$
  - H_a: $\mu > \mu_0$ Reject H_0 if $t \geq t_\alpha$ or p –value $\leq \alpha$
  - H_a: $\mu \neq \mu_0$ Reject H_0 if $t \leq -t_{\alpha/2}$ or $t \geq t_{\alpha/2}$ or p –value $\leq \alpha$

Example 1: One sample t-test

- Suppose population standard deviation of air quality is not known. Test the claim that air quality in the US is bad using a sample size of 5.

Example 2: Highway Patrol

- One-Tailed Test About a Population Mean: σ Unknown
  - At Location F, a sample of 64 vehicles shows a mean speed of 66.2 mph with a standard deviation of 4.2 mph. Use $\alpha = .05$ to test the hypothesis that average speed is within legal limit of 65 mph.

A Summary of Forms for Null and Alternative Hypotheses About a Population Proportion

- The equality part of the hypotheses always appears in the null hypothesis.
- In general, a hypothesis test about the value of a population proportion *p* must take one of the following three forms (where p_0 is the hypothesized value of the population proportion).

One-tailed (lower tail)	One-tailed (upper tail)	Two-tailed
$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$

Tests About a Population Proportion

- Test Statistic

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}}$$

where:

$$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1 - p_0)}{n}}$$

assuming $np \geq 5$ and $n(1 - p) \geq 5$

Tests About a Population Proportion

- Rejection Rule

$H_a\colon\ p > p_0$ — Reject H_0 if $z \geq z_\alpha$ or p –value $\leq \alpha$

$H_a\colon\ p < p_0$ — Reject H_0 if $z \leq -z_\alpha$ or p –value $\leq \alpha$

$H_a\colon\ p \neq p_0$ — Reject H_0 if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$ or p –value $\leq \alpha$

Example 1:

- Suppose xyz estimated a few years ago that proportion of cities with good air quality was 0.5. Recently they claim that this proportion has decreased. Test their claim using a random sample of 50 US cities.

Two-Tailed Test About a Population Proportion

- Example 2: National Safety Council

For a Christmas and New Year's week, the National Safety Council estimated that 500 people would be killed and 25,000 injured on the nation's roads. The NSC claimed that 50% of the accidents would be caused by drunk driving.

Two-Tailed Test About a Population Proportion

Example: National Safety Council

A sample of 120 accidents showed that 67 were caused by drunk driving. Use these data to test the NSC's claim with $\alpha = .05$.

Two-Tailed Test About a Population Proportion

1. Determine the hypotheses.

$$H_0: p = .5$$
$$H_a: p \neq .5$$

2. Specify the level of significance.

$$\alpha = .05$$

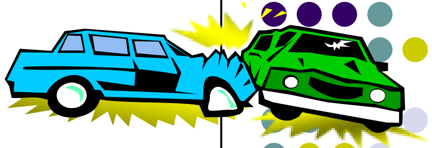
3. Compute the value of the test statistic.

$$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{.5(1-.5)}{120}} = .045644$$

a common error is using $\bar{p}$ in this formula

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}} = \frac{(67/120) - .5}{.045644} = 1.28$$

Two-Tailed Test About a Population Proportion



- p–Value Approach

4. Compute the p -value.

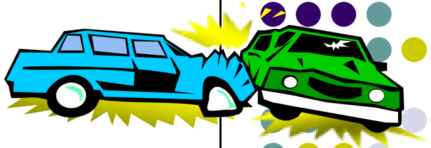
For z = 1.28, cumulative probability = .8997

p–value = 2(1 – .8997) = .2006

5. Determine whether to reject H_0.

Because p–value = .2006 > α = .05, we cannot reject H_0.

Two-Tailed Test About a Population Proportion



- Critical Value Approach

4. Determine the critical value and rejection rule.

For $\alpha/2 = .05/2 = .025$, $z_{.025} = 1.96$

Reject H_0 if $z \leq -1.96$ or $z \geq 1.96$

5. Determine whether to reject H_0.

Because $-1.96 < 1.278 < 1.96$, we cannot reject H_0.